Final Exam: Solid State Physics 476
Dec. 15, 2005

One cheat sheet, double sided. Point values are given with each question. Total exam is worth 90 points.

1. (a) How do you define a crystal structure? Are the choices you make unique? What is the difference between a primitive and non-primitive basis? (4)

   (b) Name three symmetry transformations that might apply to a crystal structure and state whether or not each one is a point group. (3)

   (c) Consider a rotation of $\pi/2$ in a tetragonal crystal about the c-axis. Describe how a point at (x, y, z) would be transformed after the rotation once you have translated it back to the unit cell. (4)

   (d) What rotations can be applied to an infinite lattice and leave it invariant (what are the angles or "foldness" of the rotations)? (3)

2. (a) Contrast the Laue picture/conditions for elastic scattering to Bragg's Law. Provide a diagram and mention reciprocal lattice vectors and show how the two agree for scattering from the (100) planes in a simple cubic lattice. (10)

   (b) If the magnitude of the wavevector of the incident radiation is 4 Å^{-1} and you observe elastic scattering from the (100) planes at $2\theta = 28\ ^{o}$ give the plane spacing. (Use whichever method you like.) (4)

   (c) Now suppose that the lattice is face-centred cubic. Why is the (100) reflection no longer observed? (2)

3. (a) State the Dulong-Petit law. What theorem forms the basis of its derivation? (4)

   (b) What is the temperature dependence of the Debye result for the specific heat of a non-conducting crystalline solid a low T? At high T? What is name given to the characteristic temperature that separates these regions? (4)

   (c) Does the contribution to the Debye specific heat arise from the acoustic phonon modes or optic modes? What model could be used to describe the other mode? (4)

4. Consider a monovalent metal such as sodium near room temperature. The conduction electron density is $n = 2.65 \times 10^{28}$ m^{-3}.

   (a) The Drude model for electrical conduction says that

   $$\sigma = \frac{ne^2}{m}\tau \qquad (1)$$

   where σ is the electrical conductivity. Calculate the effective mean free time if $e = 1.6 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg, and $\sigma = 2.11 \times 10^7\ \Omega^{-1}\text{m}^{-1}$. (3)

   (b) Calculate the Fermi energy at $T = 0$ K in electron volts. How does this compare to $k_B T$ at room temperature? (3) ($\hbar = 1.055 \times 10^{-34}$ J s, $k_B = 1.38 \times 10^{-23}$ J/K)

   (c) Give a sketch of the Fermi-Dirac distribution function for $T = 0$ K and $T > 0$ but still with E_F much greater than $k_B T$. (Sketch them on the same set of axes.) (4)

   (d) Sketch the density of states $g(E)$ for a 3-D non-interacting electron gas. (2)

(e) Answer two of the following three questions:

i. Formally, what is expression for $< E >$ the thermal average energy? (Give the integral expression and pay attention to limits.) Calculate $< E >$ at $T = 0$ K in terms of the Fermi energy. (6)

ii. In words and with a diagram (or referring to earlier diagrams) explain why the specific heat of a degenerate Fermi gas is much lower than the classical prediction. You may refer to previous answers as appropriate. What is the low-temperature temperature dependence? (6)

iii. State the Wiedemann-Franz law. Make an estimate of κ for sodium at room temperature from your previous answer assuming the Lorenz constant is $2.45 \times 10^{-8}\,\mathrm{W} \cdot \Omega/\mathrm{K}^2$. Why isn't Umklapp or impurity scattering included in this calculation? (6)

5. Please answer the following two questions dealing with bandstructure and semiconductors.

(a) What modification has to be made to the free-electron dispersion as it crosses a zone boundary? Why does this happen? Please provide a sketch which also includes the low-q dispersion. (6)

(b) The basic expression for electrical conductivity is given by equation 1. In this context why is the electrical conductivity of a pure semiconductor so low compared to that of a metal? (6)

6. Explain the difference between Type I and Type II superconductors as concerns their behaviour in a magnetic field and the difference between the London penetration depth λ_L and the Pippard coherence length ξ. Why are these two differences related to each other (i.e. what does a system try to minimize)? Please include $H-T$ (or applied-$B-T$) phase diagrams for the two types. (12)