Final Exam: Solid State Physics 476
Dec. 15, 2006

One cheat sheet, double sided. Point values are given with each question. Total exam is worth 106 points but I will mark it out of 100. Maximum mark will still be 100%. Duration: 2.5 hours.

1. Consider the space group $P\,n\,m\,a$ which is No. 62 in the crystallographic tables.

   (a) This space group is closely related to one that has 3 mirror planes that are at right angles to each other. (They share the same Bravais lattice.) Using your experience in 2-D as a guide what do you think the angles are between the a, b, and c axes if those mirror planes are present? What Bravais lattice is this? (the 3-D name is different from the 2-D name) What is the highest order rotation that is present? (e.g. 10-fold is higher than 8-fold) (5)

   (b) The P in this space group stands for *primitive*. How many lattice points are there per unit cell? (1)

   (c) The n symbol means that there is a glide plane that is perpendicular to the a direction. How do we refer to the a direction? What is the index of the plane if it is perpendicular to the a direction and intersects the a-axis at $\frac{a}{4}$? Please sketch this plane relative to the unit cell. (4)

   (d) If there was an atom at x, y, z where would it be after the glide reflection? Then, after the glide translation along the $< 011 >$ such that we move $b/2$ and $c/2$? (This is called a "net" or diagonal glide.) Perform this glide transform again on the transformed atom to show you have returned to the original location. (6)

2. (a) You perform some X-ray powder diffraction measurements on a sample and measure peaks at $2\theta = 42.63$, 49.64, and 72.83 degrees. The incident wavelength is 1.7 Å. Show that these peaks correspond to the (111), (200), and (220) peaks of a cubic lattice and at the same time find the lattice constant. Remember that in a cubic system

$$d(hkl) = \frac{a}{\sqrt{h^2 + k^2 + \ell^2}} \tag{1}$$

   (7 points)

   (b) The (100), (110), (211), and other peaks seem to be missing. What are these peaks said to be and what is the mostly likely type of cubic lattice involved? (3)

3. (a) What does the low temperature specific heat measure? Is the low temperature specific heat in solids more than, equal to, or less than that predicted by the Dulong-Petit law? (3)

   (b) What is the temperature dependence of the Debye result for the specific heat of a non-conducting crystalline solid at low T? At high T? What is name given to the characteristic temperature that separates these regions? (4)

   (c) The Einstein model, which treats each atom as an independent 3-D quantum harmonic oscillator of frequency ω, should approach the equipartition result at high temperature. The correct thermal average energy U in the Einstein model is

$$U = 3N\,\hbar\omega \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} \tag{2}$$

(There is no integral because the density of states is essentially a δ-function.) Identify the relevant terms in this equation as pertains to our class room discussions. (for example I have already mentioned what has happened to the density of states term). Show how U approaches the equipartition result for $\hbar\omega << k_B T$. (5)

4. Consider an atom of mass m that is part of a 1-dimensional chain and is connected by harmonic springs of spring constant k to its two nearest neighbours. Each atom of the chain is separated by a. We will label the atoms from the left $s-1$, s, and $s+1$. If $u_s(t)$ describes the displacement from equilibrium write the equation of motion for atom s in terms of the u's and possibly their derivatives. Under what conditions will the atom oscillate fastest? How far out of phase will the atom be from its neighbours at this point? What is the correct value for K? (or work this problem backwards if you like) Based on this assumption about the eigenvector calculate the maximum ω (you will get ω^2 equal to some multiple of k/m). (8)

5. Consider a noble metal such as copper near room temperature. The conduction electron density is $n = 8.45 \times 10^{28}$ m^{-3}.

   (a) The Drude and Sommerfeld models for the electrical conduction say that

   $$\sigma = \frac{ne^2}{m}\tau \tag{3}$$

   where σ is the electrical conductivity. Calculate the effective mean free time if $e = 1.6 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg, and $\sigma = 6.21 \times 10^7\ \Omega^{-1}\text{m}^{-1}$. (3)

   (b) Calculate the Fermi wavevector in m^{-1}. How does this compare to $\sqrt{3}\frac{\pi}{a}$ if $a = 3.61$ Å, the closest portion of the Brillouin zone. What will happen if the Fermi sphere either touches or is too close to the zone boundary? (3)

   (c) Give a sketch of the Fermi-Dirac distribution function for $T = 0$ K and $T > 0$ but still with E_F much greater than $k_B T$. (Sketch them on the same set of axes.) (4)

   (d) At the Fermi energy the density of states is $\frac{3N}{2E_F}$. Using this fact, very quickly (as I did in class) explain what the energy difference is between the two sketches you just made and estimate the specific heat by taking the derivative with respect to temperature. [Remember that if the energy range dE is small then the number of electrons in that energy range is $dE\,D(E)$.] Even if you can't make the graphical argument use something from your forumla sheet to answer the question as best you can. (6)

6. Answer 2 of the following 3 questions.

   (a) What is the difference between the Drude and the Sommerfeld models? Can either of them account for the electical conductivity in semiconductors? Explain. (6)

   (b) State the Bloch theorem and identify the terms in it. In what sense does it explain how electrons move transparently through the lattice. (it isn't the mathematical form but more what the left hand side represents). (6)

   (c) What is Umklapp scattering and where does it come from? Explain briefly how it limits the mean free path of phonons. At what temperatures is it minimized? (6)

7. (a) What is the Meissner effect? Explain what is happening at the surface of a superconductor that is exhibiting this effect. (4)

(b) Explain the difference between Type I and Type II superconductors as concerns (1) their behaviour in a magnetic field and (2) the difference between the London penetration depth λ_L and the Pippard coherence length ξ. Why are these two differences related to each other (i.e. what does a system try to minimize)? (6)

(c) What is the name of the new structure that appears in Type II superconductors in modest magnetic fields? (1)

(d) Which type of superconductor can withstand higher fields before superconductivity is destroyed. (1)

(e) Give an example of Type I superconductor? A Type II superconductor? (2)

8. I have included a figure that shows some eigenvalue solutions for the central equation applied to a 1-dimensional crystal. (You can just tear it off and include it with your answer sheet. Please put your name on it.) Please label the following features on the plot: Brillouin zone edge, valence or non-conducting band, band gap, zone centre, flattening of electron dispersion, hole region, band gap, nearly-free electron dispersion, interaction strength parameter, a dashed line indicating a Fermi energy of 10 units above 0. What kind of material would this be if the Fermi energy was 10 of the given energy units? (Assume a typical ratio between the Fermi energy and $k_B T$.) (10)

9. If a material already has disordered magnetic moments in it (belonging to the ions present) and you apply a magnetic field $\vec{H}$ do you observe paramagnetism or diamagnetism? Will the effect be greater or lesser at lower temperatures? Will electrons in a metal contribute as much, more, or less than magnetic ions? Who first developed the theory of this effect for electrons? (8)