Dept. of Philosophy St. F. X. University

BASIC CONCEPTS OF FORMAL LOGIC

Logic is one of the oldest subjects of formal instruction. It was probably taught at Plato's Academy and at other schools in ancient Greece in the fourth century B.C., and Aristotle wrote his *Prior Analytics*, the oldest surviving treatise on logic, during this period as well. The antiquity of formal logic is not surprising given that logic is the study of the correct forms of reasoning, a matter of concern to all thinking persons.

Because correct reasoning typically occurs in the context of human speech or thought, it is important not to confuse the logical aspects of speech and thought with their psychological and linguistic properties. Logic is not concerned with the mental processes that take place in our mind when we are actually thinking. Nor is logic concerned with the linguistic properties of different languages and the ways in which words come to acquire their significance in particular linguistic communities. The standards of correct reasoning established by logic are meant to apply to the evaluation of reasoning by all persons at all times and places.

Two properties of reasoning, in particular, are studied by formal logic: consistency and deductively valid inference. In order to understand consistency and valid inference, they must be distinguished from something else with which they are often confused, namely truth.

TRUTH

Truth, and its opposite, falsity, are properties that belong to only one kind of thing, propositions, or statements, because only propositions and statements make claims about the world and only claims about the world can be true or false. A proposition is true just in case the world is the way the proposition claims it to be, and false if the world is not as the proposition claims it to be. Other sentences such as commands, questions, and expressions of volition are neither true nor false because they do not make claims about the world. For example, the sentence "Pass the salt!" is in the imperative mood and is typically used to give a command; because this sentence makes no claim about the world, it is neither true nor false. For our purposes, we shall restrict our discussion of sentences to just propositions, which are capable of being either true or false.

Law of Excluded Middle: It is characteristic of propositions that they obey the Law of Excluded Middle, also known as the Law of Bivalence, which states that every proposition must be either true or false. In other words, any middle position between truth and falsity is excluded. From the Law of Excluded Middle it follows that, for any given proposition and the negation of that proposition, it must be the case that one of them is true and one false. For, if a proposition cannot be anything other than true or false, then, if a proposition is true, its negation must be false, and if a proposition is false, its negation must be true.

Law of Non-Contradiction: The propositions we are dealing with also obey the Law of Non-Contradiction, which states that it is impossible for a proposition and its negation both to be

true at the same time. In other words, one cannot truthfully both assert and deny that something is the case.

Logic is generally not able to determine whether any given proposition is true or false. In the case of some propositions, however, logic can determine their truth or falsity, namely when they are true or false already by virtue of their logical form. In particular, letting the letter P stand for any proposition, we get the following results:

1) Given the Law of Non-Contradiction, every proposition of the form "P and not P" must be false. Such a proposition is *logically false* because it is false by virtue of its logical form alone. In particular, its logical form is that of a contradiction, and, given the Law of Non-Contradiction, contradictions cannot be true.
2) Given the Law of Excluded Middle, every proposition of the form "P or not P" must be true. Such a proposition is *logically true* because it is true by virtue of its logical form alone. In particular, its logical form is that of a disjunction—two propositions joined by the word 'or'—consisting of a proposition and its negation; given the Law of Excluded Middle, it is always the case that one part of such a disjunction is true, making that disjunction as a whole to be true. Hence, a disjunction of the form "P or not P" cannot be false.
3) Every proposition that is neither logically true nor logically false is *logically indeterminate*. The logical form alone of such a proposition does not determine whether it is true or false.

CONSISTENCY

Consistency is a property of a group, or set, of propositions. A set of propositions is *consistent* if and only if it is possible for all of the propositions in that set to be true at the same time. Another way of saying this is that these propositions do not contradict one another. A set of propositions is *inconsistent* if it is not consistent, that is, if it is impossible for all the propositions in that set to be true at the same time.

Notice that the definition of consistency does not imply that all, or indeed any, of the propositions in a consistent set of propositions is in fact true; two false propositions can be consistent. To say that two propositions are consistent means simply that they both *can* be true at the same time, not that they both *are* true here and now.

DEDUCTIVE VALIDITY

Deductive validity is a property of arguments alone; it does not apply to a single proposition, or even to a set of unconnected propositions. Thus, before we can define deductive validity, we have to define what an argument is.

For the purposes of formal logic, an *argument* is defined simply as an inference claim in

which one proposition is inferred from one or more other propositions. The proposition that is inferred from the other propositions is the *conclusion* of the argument; the propositions from which the conclusion is inferred are the *premises* of the argument. Thus, in every argument there is one proposition that acts as a conclusion and one or more propositions that act as premises.

Usually, we indicate that we are making an argument by using certain words or phrases to signal that certain propositions are premises or a conclusion; these are *premise* and *conclusion indicators*. Typical premise indicators are words or phrases such as because, for, since, as, inasmuch as, it follows from, as shown by, as indicated by, the reason is that. Typical conclusion indicators are words or phrases such as hence, therefore, then, consequently, so, accordingly, it follows that, we may infer/derive/deduce that, which proves that, which shows that.

An *enthymeme* is an argument in which the conclusion or one of the premises has been left unstated.

A *sorites* is a connected series of arguments in which the conclusion of one argument also serves as a premise in another argument.

Deductive validity is the standard of good or correct deductive arguments: an argument is deductively valid if and only if whenever all the premises are true, the conclusion *must* also be true. (This kind of argument is to be contrasted with arguments in which the truth of the premises indicates only that it is *probable* that the conclusion is true; such arguments are sometimes called inductive arguments.) In other words, a deductively valid argument is one that can *never* have, at the same time, true premises and a false conclusion. Thus, to show that a particular argument is deductively valid, it is not sufficient to show that its premises and conclusion happen to be true here and now. Nor is it necessary that the conclusion or premises of a deductively valid argument actually be true. To claim that an argument is deductively valid is to say that, whatever the truth or falsity of its premises and conclusion happen to be, it could *never* be the case that, at the same time, all of the premises of that argument were true and its conclusion false. The combination of true premises and a false conclusion *never* happens in a deductively valid argument.

A *sound* argument is a deductively valid argument all of whose premises are true. Thus, the soundness of an argument requires two things: deductive validity and true premises. These two requirements are independent of each other; an argument might have true premises, but not be deductively valid, or it might be deductively valid, but not all of its premises be true.

Direct versus Indirect Proof:

The truth of a proposition can be demonstrated deductively in two ways:

1) In a *direct proof*, the conclusion of an argument is shown to be true by demonstrating that it validly follows from true premises. Such an argument is a sound argument.
2) An *indirect proof*, in contrast, involves two steps. In the first instance, an indirect proof demonstrates that a proposition is true by demonstrating that its negation is false. This step is based upon the Law of Excluded Middle, which asserts that if the negation of a proposition is false, then that proposition itself must be true. The second step in an indirect proof is showing that the negation of a proposition is false. This step is taken by showing that another, false proposition can be validly derived from the first one, that is, the second false proposition is the conclusion of a deductively valid argument that has the first proposition as its premise or one of its premises. If the conclusion of a valid argument is a false proposition, then at least one of the premises from which that conclusion has been validly derived must also be false.

   One version of an indirect argument is a *reductio ad absurdum* (reduction to absurdity) or *reductio ad impossible* (reduction to impossibility). A *reductio* (as its name is often abbreviated) is a deductively valid argument that proves that its premises, taken together as set, are inconsistent, because the conclusion of that argument is a contradiction, that is, logically false. If the conclusion of a deductively valid argument is a contradiction and always false, then it is always the case that at least one of the premises of that argument is false. If, in turn, it is always the case that at least one of the premises of an argument is false, then the premises of that argument can never all be true at the same time. Therefore, taken together, these premises constitute an inconsistent set of propositions. If all of those premises except one are true, then the remaining premise must be false. Thus, a *reductio* shows that a certain proposition is false by showing that from a set of premises that includes that proposition a contradiction can be validly derived. The contradiction in the conclusion shows that the premises are inconsistent. If all of the other propositions in an inconsistent set are true, then the remaining proposition must be false. If that proposition is false, then its negation must be true.

Deduction versus Explanation:

A deductive argument is not to be confused with a causal explanation. To explain is to give reasons that make clear *why* something is the case; this presupposes that we already know *that* something is the case. In an argument, on the other hand, it is the truth or falsity of a certain proposition that is at stake, namely that of the conclusion. In other words, an argument attempts to decide whether it is true *that* something is case. The propositions in an argument may refer to explanatory reasons, and an explanation may be stated in the form of an argument in which the premises state the causes for the state of affairs described by the conclusion, but neither is necessarily the case.

SUMMARY & EXAMPLES

Truth, Consistency, and Deductive Validity:

Truth, consistency, and deductive validity are three quite different properties, belonging to three quite different kinds of thing.

1) Truth, and its opposite, falsity, belong only to propositions, or statements.
2) Consistency, and its opposite, inconsistency, belong only to sets of propositions.
3) Deductive Validity, and it opposite, deductive invalidity, belong only to arguments, in the sense of inference claims.

Hence, only propositions can be true or false, only sets of propositions can be consistent or inconsistent, and only arguments can be deductively valid or invalid.

1) Propositions vs. Non-propositions

Proposition:	George has the suntan lotion.
Command:	George, pass the suntan lotion.
Question:	George, do you have the suntan lotion?
Request:	George, please pass the suntan lotion.

2) Sets of propositions

Consistent:	Socrates was an ancient Greek philosopher. Socrates was a citizen of ancient Athens.
Inconsistent:	Socrates was an ancient Greek philosopher who died in 399 B.C. Socrates is immortal and still alive.

3) Arguments:

Truth is neither a necessary nor a sufficient condition for deductive validity, that is, the truth of the premises and conclusion of an argument does *not* necessarily make that argument deductively valid, just as the falsity of the premises and conclusion of an argument does *not* necessarily make that argument deductively invalid.

True Premises & Conclusion, but Invalid Argument:

All Maritimers are Canadians.
<u>All Nova Scotians are Canadians</u>.
Therefore, all Nova Scotians are Maritimers.

False Premises & Conclusion, but Valid Argument:

All humans are immortal.
All dogs are humans.
Therefore, all dogs are immortal.

False Premises & True Conclusion, but Valid Argument:

All gods are mortal.
Socrates is a god.
Therefore, Socrates is mortal.

True Premises & Conclusion + Valid Argument = Sound Argument:

All men are mortal.
Socrates is a man.
Therefore, Socrates is mortal.

4) Indirect Arguments:

a) Deductively valid argument with a false conclusion:

All fish are gilled animals (animals equipped with gills).
All whales are fish.
Therefore, all whales are gilled animals.

The conclusion "All whales are gilled animals" is false. Because the argument is deductively valid, if the conclusion is false, then at least one of the premises must be false. If the premise "All fish are gilled animals" is true, then the other premise, "All whales are fish" must be false.

b) *Reductio ad absurdum* (deductively valid argument with a **logically** false conclusion):

All whales are mammals.
All mammals are animals without gills.
All whales are fish.
All fish are gilled animals.
Therefore, all whales are animals without gills and all whales are gilled animals.

The conclusion is a contradiction, a logically false proposition, and so is *always* false. Because the argument is deductively valid, it must *always* be the case that at least one of the premises is false; the premises cannot all be true together. Therefore, the premises constitute an inconsistent set of propositions.