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SYLLOGISTIC LOGIC

Syllogistic logic is the original form in which formal logic was developed; hence it is sometimes also referred to as Aristotelian logic after Aristotle, one of the founders of this branch of formal logic.

Before we can examine the logical structure of any argument to see whether it is valid or invalid, we must first examine the logical structure of the propositions out of which it is composed. In the case of syllogistic logic, this requires an examination of *categorical* propositions because all syllogistic arguments, or syllogisms, are composed of categorical propositions.

CATEGORICAL PROPOSITIONS

A categorical proposition claims that a certain relation holds between two classes, groups, or general kinds of things (a.k.a. categories). In particular, every categorical proposition claims that one class of things either includes or excludes, either in whole or in part, another class of things. As a result, every categorical proposition has a quality, either affirmative or negative, and a quantity, either universal or particular. Taken together, this means that there are four different types of categorical propositions, which appear below. Note as well that every categorical proposition is made out of exactly four components, which must appear in the following order if the proposition is to be in standard categorical form: 1) the quantifier; 2) the subject term (S); 3) the copula; and 4) the predicate term (P).

1. Universal affirmative	(A)	All S's are P's.
2. Universal negative	(E)	No S's are P's.
3. Particular affirmative	(I)	Some S's are P's.
4. Particular negative	(O)	Some S's are not P's.

The quantifier, and first word, must be either the word 'all', 'no', or 'some'. The copula is always the word 'are'.

The subject and predicate terms of a categorical proposition cannot be *singular terms*, that is, terms that refer to only one thing at a time; typical singular terms are proper names (e.g., Bob, Mary, John) or definite descriptions (e.g., the Prime Minister of Canada, the Queen of England, the tallest student in the class, the person driving this car). Rather, the terms of a categorical proposition must be *general terms*, that is, terms that can be used to refer to several members of a certain class of objects at one time. A universal proposition says something about every member of the subject class; a particular proposition says something about some members of the subject class, where 'some' is understood to mean at least one.

In addition, because the subject and predicate terms refer to classes of objects, they must both be *substantival terms* (i.e., nouns or expressions containing at least one noun) and not *attributive terms* (i.e., adjectives or adjective phrases).

Sentences that are not yet in standard form can be transformed, or translated, into standard form by various devices, including the substituting of substantival phrases for adjectival ones, supplying a quantifier when none is explicitly stated, and using the predicate term to express the action contained in the verb when the latter is not the verb 'are'.

Venn Diagrams

Venn Diagrams provide a useful way of representing pictorially what is being claimed about the relation between the subject and predicate classes in a categorical proposition. Each diagram consists of two overlapping circles, one representing the subject class and the other the predicate class. By making the two circles overlap, four distinct areas are created:

1) the area inside the S circle, but outside the P circle; 2) the area of intersection between the two circles; 3) the area inside the P circle, but outside the S circle; 4) the area outside both circles.

With respect to each one of these areas, one can either (a) leave it blank, which makes no statement about what is inside it, or (b) shade it in, which tells us that it is empty, or (c) put an asterisk in it, which tells us that there is at least one thing in that area. Each of the four standard categorical propositions is represented by a different Venn diagram.

Distribution of Terms

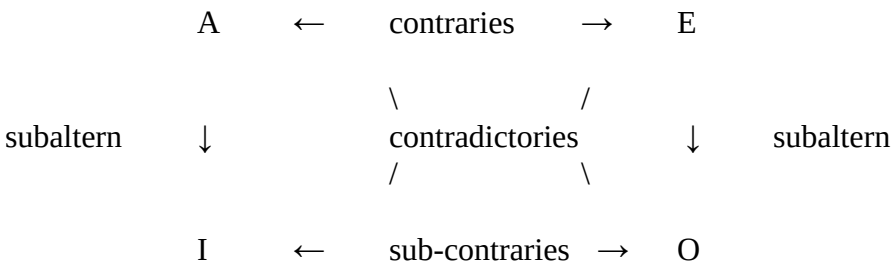
The terms of a categorical proposition, whether subject or predicate, are said to be distributed when a claim is being made about *every* member of the class of objects represented by that term. A-form propositions distribute only the subject term; E-form propositions distribute both the subject and the predicate term; I-form propositions distribute neither term; and O-form propositions distribute only the predicate term. A useful mnemonic device to help one remember this pattern is the formula that universal propositions distribute the subject term and negative propositions distribute the predicate term.

LOGICAL RELATIONS BETWEEN CATEGORICAL PROPOSITIONS AND IMMEDIATE INFERENCE

There are several logical relations between categorical propositions having the same subject and predicate terms or the negation of the same terms. In certain instances, these relations allow us to make valid inferences from one such categorical proposition to another by means of a process known as *immediate inference*, that is, an inference which does not require a mediating term or proposition.

The Traditional Square of Opposition

The square of opposition shows the different logical relations between pairs of categorical propositions that have the same subject term and the same predicate term, but different quantities and qualities.



Contradictory Relation:

Contradictory propositions can never both be true at the same time nor both false at the same time; if one is true, the other is false, and if one is false, the other is true. The "A" and the "O" propositions are contradictories, that is, each is the contradictory of the other. Given the truth of an "A" proposition, we can validly infer the falsity of the corresponding "O" proposition and *vice versa*, and given the falsity of the "A" proposition, we can validly infer the truth of the corresponding "O" proposition and *vice versa*. The same relation obtains between the "E" and "I" propositions.

Contrary Relation:

Contrary propositions can never both be true at the same time, but they can both be false. The "A" and "E" propositions are contraries. Given the truth of one of them, we can validly infer the falsity of the other, but given the falsity of either the "A" or the "E" proposition, no valid inference with respect to the truth or falsity of its contrary is possible.

Sub-contrary Relation:

Sub-contrary propositions can both be true, but they cannot both be false; one or the other of them must be true. The "I" and "O" propositions are sub-contraries. Given the falsity of one of them, we can validly infer the truth of the other, but given the truth of an "I" or "O" proposition, no valid inference with respect to the truth or falsity of its sub-contrary is possible.

Subaltern Relation:

A particular proposition ("I" or "O") is said to be the subaltern of its corresponding universal proposition (i.e., the "A" or "E" proposition, respectively) when the truth of the universal proposition implies the truth of the corresponding particular proposition. Given the truth of an "A" proposition, we can validly infer the truth of the corresponding "I" proposition, and given the falsity of an "I" proposition, we can validly infer the falsity of the corresponding "A" proposition. But given the truth of an "I" proposition, we cannot validly infer the truth of the corresponding "A" proposition, and given the falsity of an "A" proposition, we cannot validly infer the falsity of the corresponding "I" proposition. The same relation holds between "E" and "O" propositions.

Remember that the logical relations set out in the square of opposition only hold of categorical propositions with the same subject and predicate terms; an "A" and an "O" proposition, for example, with different subject or predicate terms are not contradictories.

Immediate Inference and Logical Equivalence

In addition to the logical relations set out in the square of opposition, there are the following relations of immediate inference which are based on the notion of logical equivalence (two or more propositions are logically equivalent if they are true or false under exactly the same conditions):

Converse Relation:

To perform conversion, one simply switches the positions of the subject and predicate terms. This does not, however, yield in all cases a proposition which is logically equivalent to the original proposition; in order for conversion to yield a valid inference, no term may be distributed in the converse which was not distributed in the original proposition.

The converse of SeP is PeS.
The converse of SiP is PiS.
The converse of SaP is PaS. (Not logically equivalent)
The converse of SoP is PoS. (Not logically equivalent)

Using *conversion by limitation*, one can validly infer an "I" proposition (PiS) from an "A"

proposition (SaP). Notice that the resulting "I" proposition is not logically equivalent to the original "A" proposition.

Obverse Relation:

To perform obversion, one switches the quality of the proposition and substitutes for the original predicate its contradictory. If "P" is the original predicate, its contradictory is "non-P". The resulting proposition is logically equivalent to the original proposition in all four cases.

The obverse of SaP is Se non-P.
The obverse of SeP is Sa non-P.
The obverse of SiP is So non-P.
The obverse of SoP is Si non-P.

Contrapositive Relation:

To perform contraposition, one switches or transposes the subject and predicate terms and substitutes for each its respective contradictory. The same result can be reached by performing first obversion, then conversion, and finally obversion again.

The contrapositive of SaP is non-P a non-S.
The contrapositive of SoP is non-P o non-S.
The contrapositive of SeP is non-P e non-S. (Not equivalent)
The contrapositive of SiP is non-P i non-S. (Not equivalent)

THE SYLLOGISM (MEDIATE INFERENCE)

Definition: A syllogism is an argument composed of three categorical propositions, in which, given two propositions (the premises), which have one term in common, we infer a third proposition (the conclusion) whose terms are found one in each of the two premises.

Thus, in a syllogism an inference is made about the relation between the two terms in the conclusion on the basis of the relations between these two terms, respectively, and some other, third term. It is called a mediate inference because the third term is said to mediate between or connect the other two terms.

Terminology and Symbolism

The *major* term is the predicate term of the conclusion. Symbol: "P"

The *minor* term is the subject term of the conclusion. Symbol: "S"

The *middle* term is the term found in both premises, but not in the conclusion. Symbol: "M"

The *major* premise is the premise in which the major term occurs, whether as the subject or the predicate term of that premise. It is generally written first of the two premises when the argument is stated in standard form. The *minor* premise is the premise in which the minor term occurs, whether as the subject or the predicate term of *that* premise. Hence, the major premise consists of the major and middle terms (in either order), and the minor premise consists of the minor and middle terms (in either order).

Syllogisms are distinguished by (a) their *figure*, which is determined by the position of the middle term in the premises, and (b) their *mood*, which is determined by the quantity and quality of the propositions which act as the premises and conclusion of the argument.

There are 256 different possible syllogisms; all but 24 are invalid. As logicians, we are interested in distinguishing the valid from the invalid forms of syllogistic reasoning. We shall consider two ways of doing this: 1) by means of a set of formal rules; 2) by means of a Venn diagram.

Testing for Validity: Rules of the Syllogism

A syllogism is invalid if it breaks one or more of the following rules:

1. The middle term must be distributed at least once. Otherwise, the *fallacy of the undistributed middle* has occurred.

2. No term may be distributed in the conclusion if it is not distributed in the premise in which it occurs. Otherwise, the *fallacy of illicit process of the major (or minor) term* has occurred.

3. At least one premise must be affirmative.

4. If one premise is negative, the conclusion must be negative, and *vice versa*.

5. If *both* premises are universal and the conclusion is particular, the syllogism is valid only from an existential point of view and not from a hypothetical one.

The following mnemonic verse, which goes back to William Shyreswood, a thirteen-century English logician, lists the valid moods in each figure:

Barbara Celarent Darii Ferioque prioris;
Cesare Camestres Festino Baroco secundae;
Tertia Darapti Disamis Datisi Felapton
Bocardo Ferison habet; quarta insuper addit
Bramantip Camenes Dimaris Fesapo Fresison.

Testing for Validity: Venn Diagram

This is done by representing the two premises on a Venn diagram consisting of three circles, one for each term in the syllogism. An argument is valid if and only if, in so representing the premises, the conclusion has thereby also been represented on the diagram. If it has not, the argument is invalid.

Given an example of syllogistic inference in ordinary English, one should:

a) Determine the conclusion and its logical form.
b) Determine the premises and their logical form.
c) Write out the premises and conclusion in standard categorical form and order (major premise, minor premise, and conclusion).
d) Symbolize the argument.
e) Test its validity by applying the rules of the syllogism or a Venn diagram.