

\*Your lowest assignment mark will not be used to calculate your average assignment mark. So if you've handed in all the assignments (#1,2,3,4,5,6,7), and are happy with the results, a zero mark for assignment #8 will not affect your course mark.

**Q1.** Calcium oxide is produced on a large scale by the decomposition of the mineral calcite ( $\text{CaCO}_3$ ).



- a) The equilibrium constant for this reaction is  $K = p_{\text{CO}_2}$ . What about the  $\text{CaCO}_3(\text{s})$  and  $\text{CaO}(\text{s})$ ? Are these compounds ignored in the equilibrium? Explain briefly.

[3] b) Calculate  $K$  at 25 °C.

- c) Calculate the decomposition temperature of calcite at 1.00 bar pressure. Hint: Calculate the temperature at which the equilibrium  $\text{CO}_2$  pressure is 1.00 bar using  $d\ln K/d(1/T) = -\Delta H^\circ/R$ .

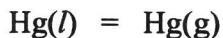
Data at 25 °C:	$\Delta G_{\text{fm}}^\circ(\text{CaCO}_3, \text{s}) = -1128.8 \text{ kJ mol}^{-1}$	$\Delta H_{\text{m}}^\circ(\text{CaCO}_3, \text{s}) = -1206.9 \text{ kJ mol}^{-1}$
	$\Delta G_{\text{fm}}^\circ(\text{CaO}, \text{s}) = -604.0 \text{ kJ mol}^{-1}$	$\Delta H_{\text{fm}}^\circ(\text{CaO}, \text{s}) = -635.1 \text{ kJ mol}^{-1}$
	$\Delta G_{\text{fm}}^\circ(\text{CO}_2, \text{g}) = -394.4 \text{ kJ mol}^{-1}$	$\Delta H_{\text{fm}}^\circ(\text{CO}_2, \text{g}) = -393.5 \text{ kJ mol}^{-1}$

**Q2.** The  $\text{CaCO}_3(\text{s}) = \text{CaO}(\text{s}) + \text{CO}_2(\text{g})$  reaction is the basis of an industry valued at about 700 billion dollars annually. Name the product that is manufactured and its main uses.

**Q3.** A proposal is made to reduce atmospheric carbon dioxide levels by using CaO to absorb  $\text{CO}_2$ . Is [1] this proposal feasible? Explain briefly.

**Q4.** Standard Gibbs energies of formation are widely used to calculate equilibrium constants for chemical reactions. This question shows  $\Delta G_f^\circ$  data can also be used to predict **phase equilibria**.

- a) Use the data given below to calculate the vapor pressure of liquid mercury at 25 °C.



[4] b) Also use the data to calculate the boiling point of liquid mercury at 1.00 bar.

- c) The measured boiling point of liquid mercury (357 °C) is slightly different from the value calculated in part b. Why?

Data at 25 °C: $\Delta G_{\text{fm}}^\circ(\text{Hg}, \text{g}) = 31.82 \text{ kJ mol}^{-1}$	$\Delta H_{\text{m}}^\circ(\text{Hg}, \text{g}) = 61.32 \text{ kJ mol}^{-1}$
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**Q5.** This question refers to the reduced van der Waals equation:

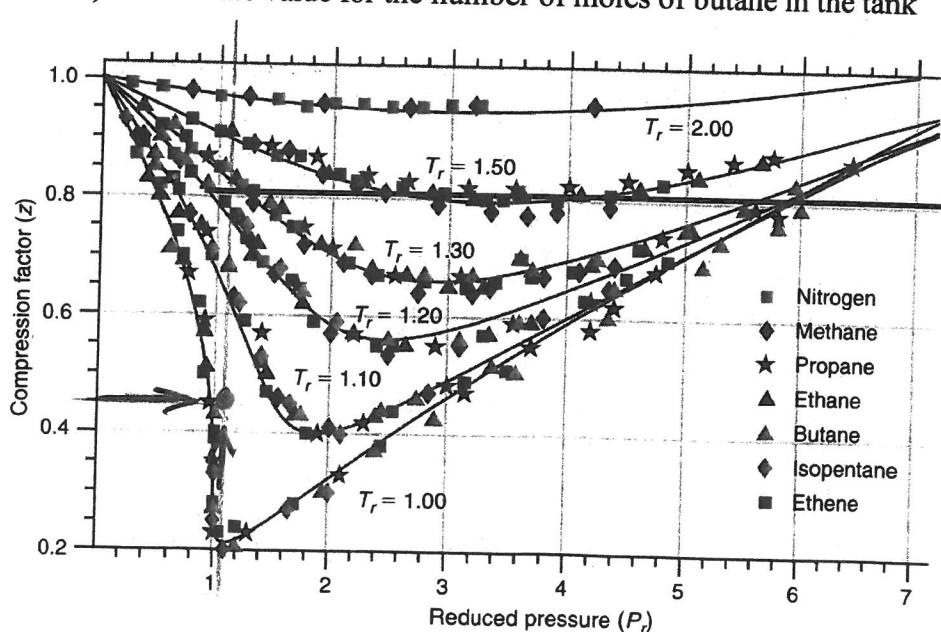
$$p_r = \frac{8}{3} \frac{T_r}{V_r - \frac{1}{3}} - \frac{3}{V_r^2}$$

- a) This is a “universal” equation of state. Why?
- b) Use the equation to calculate the reduced compression factor  $Z_r$  at the critical point.
- [6] c) Use the equation to show the reduced isothermal compressibility  $-V_r^{-1}(\partial V_r / \partial p_r)_T$  is infinite at the critical point.
- d) What does “reduced” mean in this context?
- e) Show the reduced isothermal compressibility  $\kappa_r = -V_r^{-1}(\partial V_r / \partial p_r)_T$  is infinite at the critical point! *Suggestion:*  $(\partial V_r / \partial p_r)_T$  is difficult to evaluate. Instead, evaluate  $(\partial p_r / \partial V_r)_T$  and use the inverse rule.

**Q6.** Some models of paint-ball guns use capsules of compressed carbon dioxide to fire projectiles with muzzle velocities of about 100 meters per second. After firing a few shots, the carbon dioxide pressure might be expected to drop, causing lower velocities and poor accuracy. Yet consistent muzzle velocities are maintained until the capsules are almost empty. Explain.

**Q7.** A 50.0 L tank contains butane ( $T_c = 425$  K,  $p_c = 38.0$  bar) at 450 K and 42.0 bar.

- [3] a) Calculate the number of moles of butane in the tank using the ideal gas law. Why is this calculation likely to be inaccurate?
- b) Use the corresponding-states graph given below to calculate:
  - i) the compression factor  $Z$
  - ii) an accurate value for the number of moles of butane in the tank



(Q1)



a) equilibrium constant  $K = P_{\text{CO}_2}$  only  $P_{\text{CO}_2}$ ?

are  $\text{CaCO}_3(s)$  and  $\text{CaO}(s)$  being ignored?

$$\text{No! } K = e^{-\Delta G^\circ/RT} = e^{-[\Delta G_f^\circ(\text{CaO}) + \Delta G_f^\circ(\text{CO}_2) - \Delta G_f^\circ(\text{CaCO}_3)]/RT}$$

but  $\text{CaCO}_3(s)$  and  $\text{CaO}(s)$  have such small molar volumes that  $G^\circ(\text{CaCO}_3, s)$  and  $G^\circ(\text{CaO}, s)$  are virtually independent of the pressure and therefore equal to the standard values  $G_f^\circ(\text{CaCO}_3, s)$  and  $G_f^\circ(\text{CaO}, s)$

$$\begin{aligned} b) \quad \Delta G^\circ &= \Delta G_f^\circ(\text{CaO}, s) + \Delta G_f^\circ(\text{CO}_2, g) - \Delta G_f^\circ(\text{CaCO}_3, s) \\ &= [-604.0 - 394.4 - (-1128.8)] \text{ kJ mol}^{-1} \end{aligned}$$

$$\Delta G^\circ = 130.4 \text{ kJ mol}^{-1}$$

$$K = \exp\left(-\frac{\Delta G^\circ}{RT}\right) = \exp\left[-\frac{130400 \text{ J mol}^{-1}}{8.314 \text{ J K}^{-1} \text{ mol}^{-1} (298.15 \text{ K})}\right]$$

$$\begin{aligned} K &= e^{-52.60} \\ &= \boxed{1.424 \times 10^{-23} \text{ bar}} \end{aligned}$$

$$\begin{aligned} c) \quad \Delta H^\circ &= \Delta H_f^\circ(\text{CaO}, s) + \Delta H_f^\circ(\text{CO}_2, g) - \Delta H_f^\circ(\text{CaCO}_3, s) \\ &= -635.1 - 393.5 - (-1206.9) \end{aligned}$$

$$\Delta H^\circ = 178.3 \text{ kJ} \quad (\text{endothermic})$$

(Q1 c) cont.)

van't Hoff equation  $\frac{d \ln K}{dT} = -\frac{\Delta H^\circ}{R}$

gives  $d \ln K = -\frac{\Delta H^\circ}{R} d \frac{1}{T}$

integrate from  $T_1$  to  $T_2$   $\int_{\ln K(T_1)}^{\ln K(T_2)} d \ln K = -\frac{\Delta H^\circ}{R} \int_{T_1}^{T_2} d \frac{1}{T}$  (assuming  $\Delta H^\circ$  is constant)

$$\ln K_2 - \ln K_1 = -\frac{\Delta H^\circ}{R} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

at  $T_1 = 298.15 \text{ K}$ ,  $K_1 = 1.424 \times 10^{-23} = P_{\text{CO}_2}$  at  $T_1$

at  $T_2 = ?$ ,  $K_2 = 1.00 \text{ bar} = P_{\text{CO}_2}$  at  $T_2$

$$\ln(1) - \ln(1.424 \times 10^{-23}) = -\frac{178300 \text{ J mol}^{-1}}{8.314 \text{ J K}^{-1} \text{ mol}^{-1}} \left( \frac{1}{T_2} - \frac{1}{298.15 \text{ K}} \right)$$

solve for  $T_2 = 1110 \text{ K}$

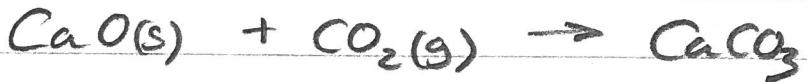
this is the decomposition temp. of  $\text{CaCO}_3(s)$  at 1.00 bar

(Q2)  $\text{CaO}$  along with clay minerals (aluminosilicates) and gypsum is used to make cement

mixing cement with water and "aggregate" (sand or gravel) makes concrete — essential construction material for foundations, roads, pipes, floors, ...

Q3

calcium oxide is very efficient at absorbing carbon dioxide gas :



$$K = \frac{1}{P_{\text{CO}_2}} = 7.02 \times 10^{22} \text{ bar}^{-1} \text{ at } 25^\circ\text{C}$$

(from Q1)

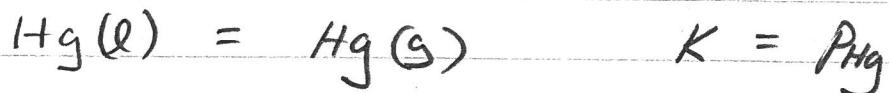
but removing atmospheric  $\text{CO}_2$  using  $\text{CaO}$  is not feasible!

an equal amount of atmospheric  $\text{CO}_2$  is released when  $\text{CaO}$  is manufactured ( $\text{CaCO}_3 \rightarrow \text{CaO} + \text{CO}_2$ )!

also, large amounts of fossil fuels (oil, coal or natural gas) must be burned to heat  $\text{CaCO}_3$  to its decomposition temperature, releasing more atmospheric  $\text{CO}_2$ !

Q4

a) liquid-vapor equilibrium for mercury:



$$\Delta G^\circ = \Delta G_{\text{f,m}}^\circ(\text{Hg, g}) - \Delta G_{\text{f,m}}^\circ(\text{Hg, l})$$

$$\Delta G^\circ = 31820 \text{ J mol}^{-1} - 0 \quad \begin{array}{l} \text{(an element, pure)} \\ \text{and in its most} \\ \text{stable state} \end{array}$$

$\therefore \Delta G_f^\circ = 0$

$$K = e^{-\Delta G^\circ/RT} = e^{-31820/(8.314)(298.15)}$$
$$= e^{-12.84} = 2.66 \times 10^{-6} \text{ bar} \quad (\text{at } 25^\circ\text{C})$$

(Q4 cont.)

b) at  $T_1 = 298.15 \text{ K}$ ,  $K_1 = 2.661 \times 10^{-6} \text{ bar}$

at  $T_2 = ?$ ,  $K_2 = 1.00 \text{ bar}$

(from Q1:)  $\left[ \Delta H^\circ = \Delta H_{\text{fus}}^\circ(\text{Hg, g}) - \cancel{\Delta H_{\text{fus}}^\circ(\text{Hg, l})} = 61320 \text{ J mol}^{-1} \right]$

$$\ln K_2 - \ln K_1 = -\frac{\Delta H^\circ}{R} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$\ln(1.00) - \ln(2.661 \times 10^{-6}) = -\frac{61320 \text{ J mol}^{-1}}{8.314 \text{ J K}^{-1} \text{ mol}^{-1}} \left( \frac{1}{T_2} - \frac{1}{298.15 \text{ K}} \right)$$

Solve for  $T_2 = 620 \text{ K}$  (346 °C)

c) the actual (measured) boiling point of mercury at 1.00 bar is 357 °C, slightly different from the value calculated in part b)

Why? the calculation assumed  $\Delta H^\circ$  is constant from 25 °C to 346 °C, when in fact  $\Delta H^\circ$  decreases slightly with increasing T

Q5

a) the reduced van der waals equation

$$P_r = \frac{8}{3} \frac{T_r}{V_r - \frac{1}{3}} - \frac{3}{V_r^2} \quad \left( \begin{array}{l} \text{in the limit } V_r \rightarrow \infty \\ (\rho \rightarrow 0), \text{ the ideal gas} \\ \text{equation is } P_r = \frac{8T_r}{3V_r} \end{array} \right)$$

is a "universal" equation of state because  
 $P_r, T_r, V_r$  are "natural" quantities (no arbitrariness  
defined "human" units such as bar, K, L)  
and there are no arbitrarily defined constants  
(such as  $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$   $\Rightarrow$  human-defined)

b) at the critical point ( $T=T_c, \rho=\rho_c, V_m=V_{mc}$ )  
the reduced temperature, pressure, and molar volume  
are:

$$T_r = \frac{T}{T_c} = \frac{T_c}{T_c} = 1 \quad P_r = \frac{P}{P_c} = \frac{\rho_c}{\rho_c} = 1 \quad V_r = \frac{V_m}{V_{mc}} = \frac{V_{mc}}{V_{mc}} = 1$$

and the reduced compression factor is

$$Z_r = \frac{\text{Practical}}{\text{Pideal}} = \frac{\text{Practical}}{\frac{8}{3} \frac{T_r}{V_r}} = \frac{(1)}{\frac{8}{3}(1)} = \boxed{\frac{3}{8}}$$

$$c) \quad P_r = \frac{8}{3} \frac{T_r}{V_r - \frac{1}{3}} - \frac{3}{V_r^2}$$

$$\left( \frac{\partial P_r}{\partial V_r} \right)_{T_r} = \frac{8}{3} T_r \frac{\partial}{\partial V_r} \left( V_r - \frac{1}{3} \right)^{-1} - 3 \frac{\partial V_r^{-2}}{\partial V_r}$$

$$= \frac{8}{3} T_r \frac{(-1)}{\left( V_r - \frac{1}{3} \right)^2} - 3 \frac{(-2)}{V_r^3} = -\frac{8}{3} \frac{1}{\left( \frac{2}{3} \right)^2} + \frac{6}{1} = 0$$

( Q5 cont.)

- d) reduced means divided by the corresponding value at the critical point

(Q6)

the capsules contain compressed  $\text{CO}_2$  vapor and liquid at the boiling point

after each shot, some  $\text{CO}_2(l)$  evaporates to maintain a steady pressure of  $\text{CO}_2(g)$  in the capsule

(Q7)

a)  $n = \frac{PV}{RT} = \frac{(42.0 \text{ bar})(50.0 \text{ L})}{(0.08314 \frac{\text{L bar}}{\text{K mol}})(450\text{K})} = 56.1 \text{ mol}$

b) reduced pressure  $P_r = \frac{P}{P_c} = \frac{42.0 \text{ bar}}{38.0 \text{ bar}} = 1.05$

reduced temperature  $T_r = \frac{T}{T_c} = \frac{450\text{K}}{425\text{K}} = 1.059$

i) read  $Z \approx 0.45$  on the  $Z$  vs  $P_r$  graph

ii)  $Z = \frac{PV}{nRT} \quad n = \frac{1}{Z} \frac{PV}{RT} = \frac{1}{0.45} (56.1 \text{ mol})$

$n = 125 \text{ mol}$