Thermal Radiation

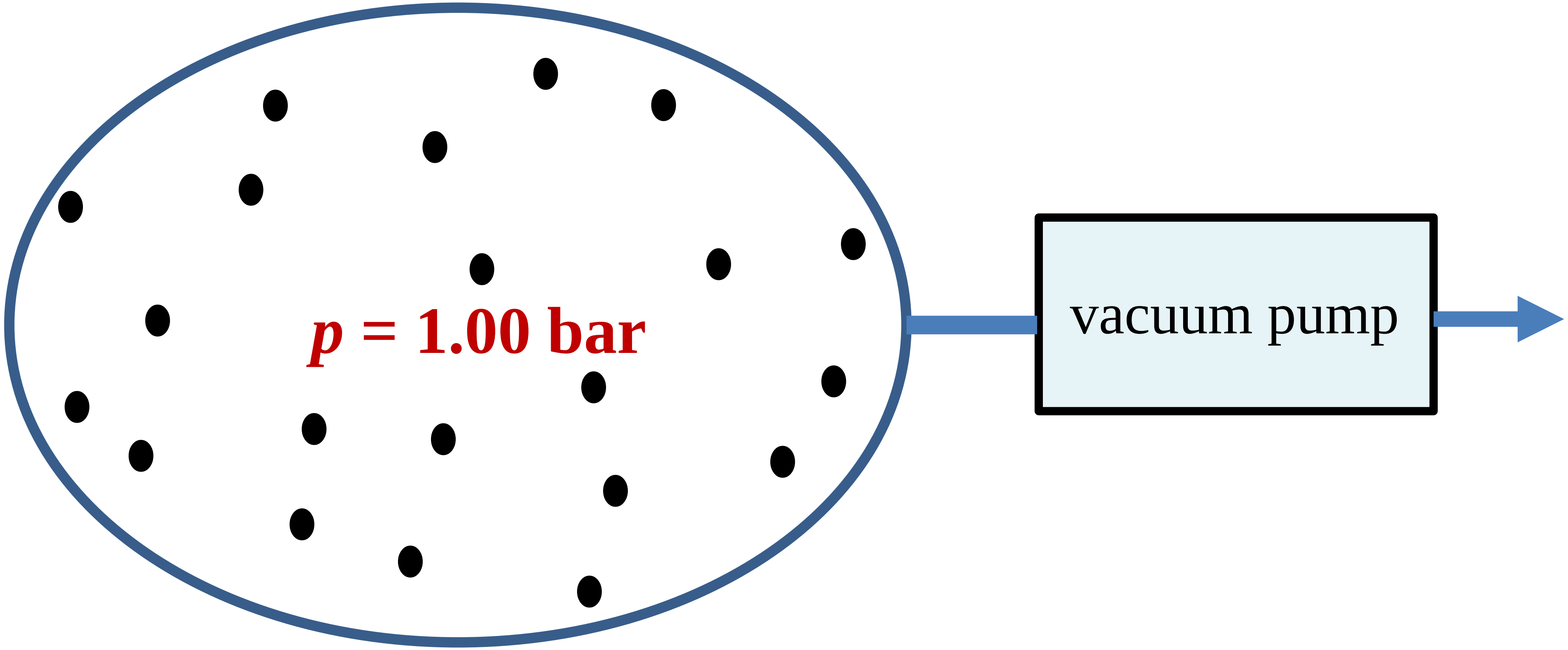
All objects emit electromagnetic energy as as thermal radiation, **a fundamental but often often overlooked property of matter**.

Thermal radiation has important applications related to radiant heating and cooling, remote sensing, infrared thermometers, the greenhouse effect, thermonuclear reactions, and the evolution of stars.

Thermodynamics of Radiation – Photon Gas

Thermodynamics applies to many systems, even to ***non-material systems***!

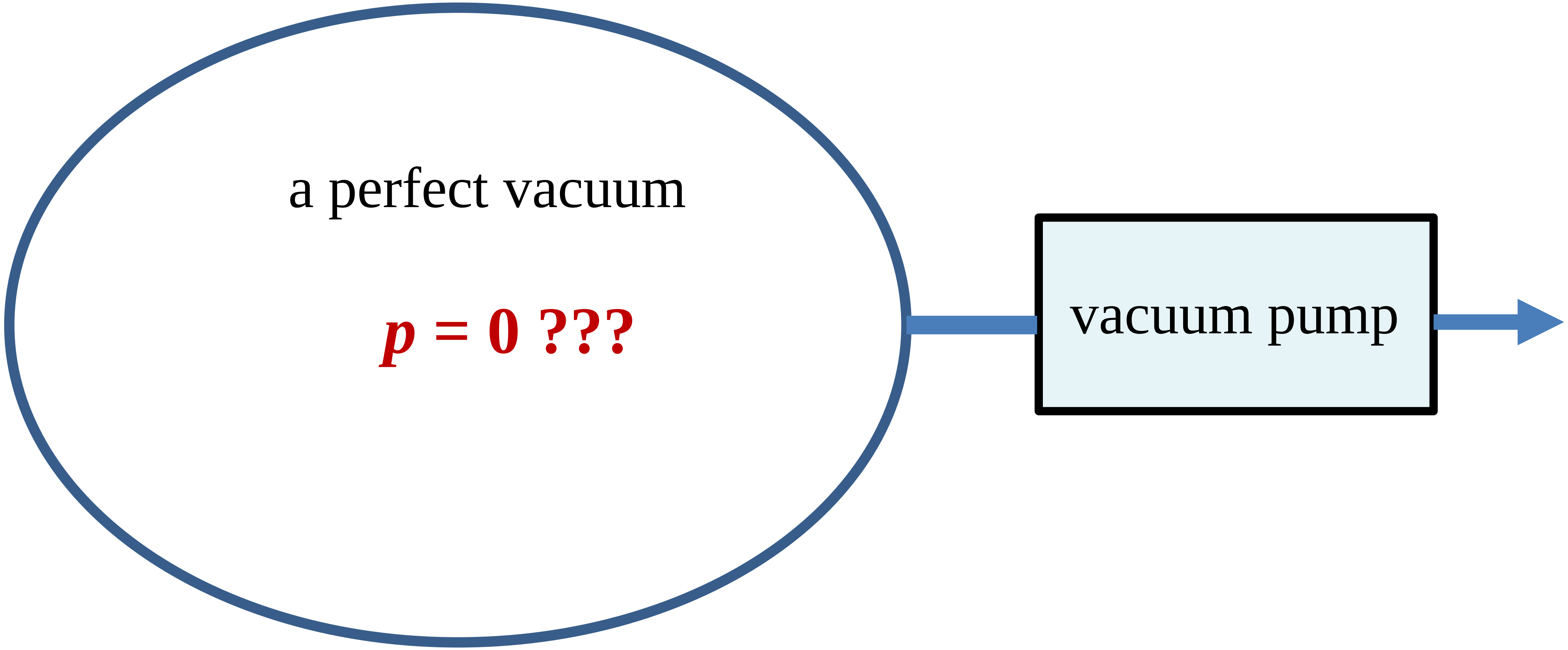
Suppose a "super" vacuum pump is attached to a tank filled with air.



Every gas molecule is pumped out of the tank.

$n = 0$ $p = nRT/V = 0$

But is the tank empty? **Is the pressure in the tank zero?**



a perfect vacuum

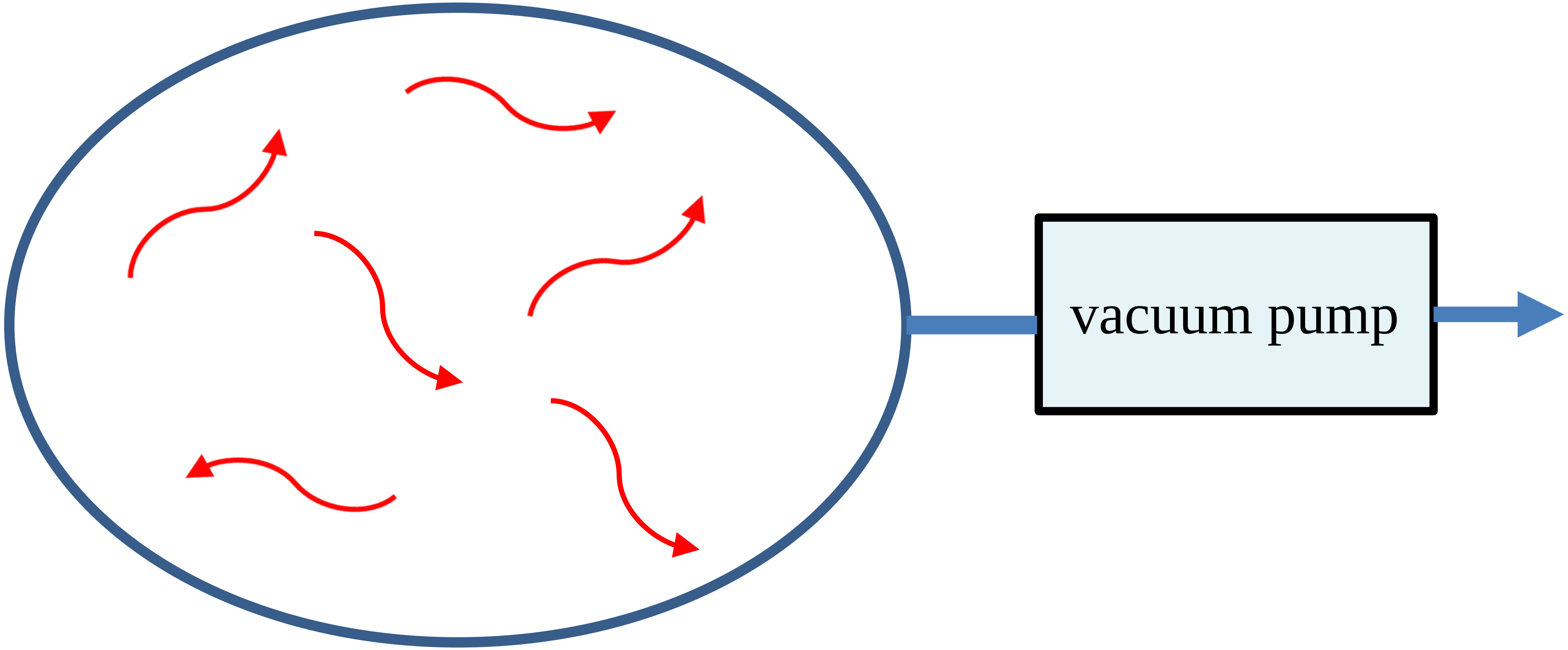
$p = 0$???

vacuum pump

All materials, including the walls of the tank, emit **thermal radiation**.

The radiation moves at the speed of light (2.998 x 10^9 m s^{-1}), a very high but not infinite speed, so it takes a few ns for the radiation to cross the tank.

Conclusion: **the tank is filled with thermal radiation** (photons in transit)



Thermodynamics of Radiation

Helmholtz function

$$\mathrm{d}A = -S\mathrm{d}T - p\mathrm{d}V$$

$(A = U - TS)$

$$= \left(\frac{\partial A}{\partial T}\right)_V \mathrm{d}T + \left(\frac{\partial A}{\partial V}\right)_T \mathrm{d}V$$

first derivatives

$$-S = \left(\frac{\partial A}{\partial T}\right)_l \qquad -p = \left(\frac{\partial A}{\partial V}\right)_T$$

Maxwell relation

$$\left[\frac{\partial}{\partial V}\left(\frac{\partial A}{\partial T}\right)_V\right]_T = \left[\frac{\partial}{\partial T}\left(\frac{\partial A}{\partial V}\right)_T\right]_V$$

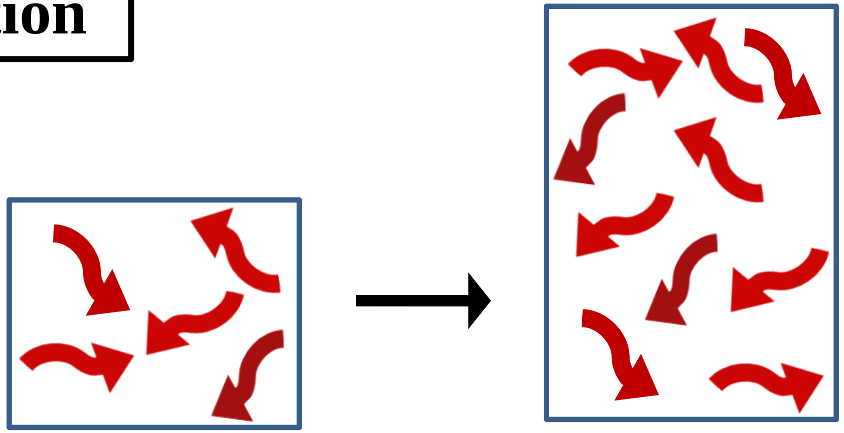
gives:

$$\boxed{\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V}$$

So what?

Thermodynamics of Radiation

$$\boxed{\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V}$$



Significance: Increasing the volume of radiation at constant temperature produces a larger number of photons and a higher entropy, so $(\partial S / \partial V)_T > 0$.

Then $(\partial p / \partial T)_V > 0$ from the Maxwell relation.

Conclusion: **Radiation exerts a pressure that increases with temperature.**

Thermodynamics of Radiation

h = Planck constant
v = radiation frequency
U = energy
c = speed of light
$u = U/V$ = energy density

To develop an equation of state for radiation, consider N photons of average energy hv in a cubic box with sides of length L and box volume $V = L^3$.

At a given instant, one-sixth of the photons are moving towards one of the walls.

The average momentum of a photon is hv/c. After collision with a wall and reflection, the photon momentum is $-hv/c$, so the change in momentum per collision is $-2hv/c$.

In the time dt, one sixth of the photons in the volume $L^2c\mathrm{d}t$ near a wall will hit the wall.

The pressure on the wall is the rate of change of momentum per unit area:

$$p = \frac{1}{6}N\frac{L^2c\mathrm{d}t}{V}\frac{2hv}{c}\frac{1}{\mathrm{d}t} = \frac{1}{3}\frac{Nhv}{V} = \frac{1}{3}\frac{U}{V} = \frac{1}{3}u = \text{one third of the energy density}$$

Thermodynamics of Radiation

thermodynamic equation of state

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

use $u = U/V$ and $p = u/3$

$$u(T) = \frac{T}{3}\left(\frac{\partial u(T)}{\partial T}\right)_V - \frac{u(T)}{3}$$

$$4u(T) = T\,\frac{\mathrm{d}u(T)}{\mathrm{d}T}$$

integrates to give

$$\boxed{u(T) = U(T)/V = \beta T^4}$$

$$\boxed{p(T) = \beta T^4/3}$$

Radiation pressure is proportional to the temperature *raised to the fourth power*!

$$\boxed{\text{radiation constant}\ \ \beta = 7.56 \times 10^{-21}\,\text{bar K}^{-4}}$$

Thermal Radiation Pressure

- depends only on the temperature
- negligible at room temperature (6.12 x 10^{-11} bar at 300 K)
- but increases very rapidly with temperature (*e.g.*, 7560 bar at 10^6 K)
- completely independent of the material emitting the radiation
- continuous spectrum, not quantized like atomic and molecular spectra

Thermal Radiation Pressure - a few applications

1. **Stellar evolution** depends crucially on thermal radiation pressure.

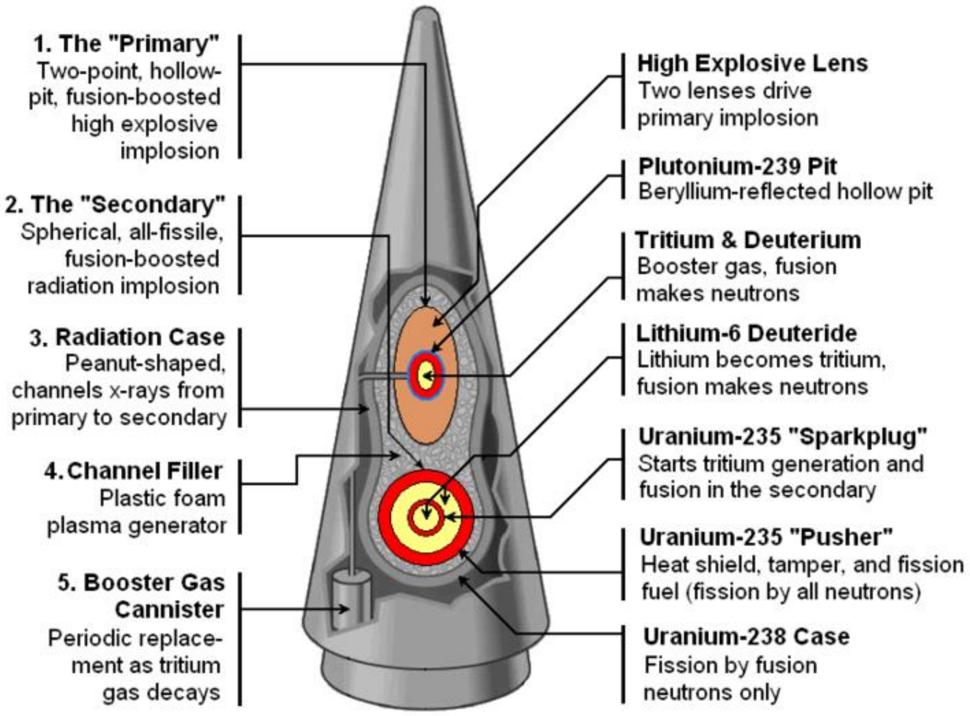
Radiation pressure keeps large stars from collapsing under their own weight, until they eventually run out of nuclear fuel and implode, sometimes forming black holes.

The universe is filled with **cosmic microwave background radiation** equivalent to thermal radiation emitted by an object at 2.8 K.

This radiation is the cooled and expanded photon gas from the primordial fireball at the “big bang” birth of the universe.

Thermal Radiation Pressure - a few applications

2. Megaton TNT-equivalent **thermonuclear weapons** for WW III are powered the fusion of hydrogen and lithium isotopes driven by the intense **radiation pressure** from kiloton-scale nuclear fission detonations of uranium or plutonium isotopes at **10^7 K**.



W88 Warhead for Trident D-5 Ballistic Missile

Thermal Radiation Pressure - a few applications

3. Multi-billion-dollar international research projects are developing **controlled thermonuclear fusion reactions** such as

$$\mathbf{{}^{2}H + {}^{3}H \rightarrow {}^{4}He + neutron} \qquad \Delta H = 1.67\ \underline{\textbf{billion}}\ \text{kJ per mole!}$$

to provide virtually unlimited energy for humankind.

At the **National Ignition Facility** (NIF) in California, high-power lasers "blast" small capsules of nuclear fuel. The high radiation pressures compress and heat the fuel to initiate controlled thermonuclear, fusion, releasing heat to produce electricity.