

- A 0.500 L container is filled with argon at a pressure of 0.200 bar at 300 K. Calculate:

 - the number of argon-argon collisions per second in the container (use $\sigma = 0.36 \text{ nm}^2$)
 - the number of argon atom collision with the container wall per cm^2 per second.
- A 0.500 L container is filled with H atoms at a pressure of 0.200 bar and Cl atoms at a pressure of 0.300 bar at 1000 K. Calculate the number of H-Cl collisions per second in the container. Use $r_{\text{H}} = 0.10 \text{ nm}$ and $r_{\text{Cl}} = 0.17 \text{ nm}$ for the collision radii.
 - Use the answer from part a to place an upper limit on molar rate constant $k_{2\text{M}}$ for the reaction



- Give two reasons why you'd expect helium to have a larger diffusion coefficient than argon at a given temperature and pressure. (No calculations required!)
- Explain why effusion can be used to separate isotopes.
- In a diffusion experiment, N_0 tracer molecules are initially located at position $x = 0$ in a tube of cross-sectional area A . After diffusion for time t , the concentration of molecules along the tube is

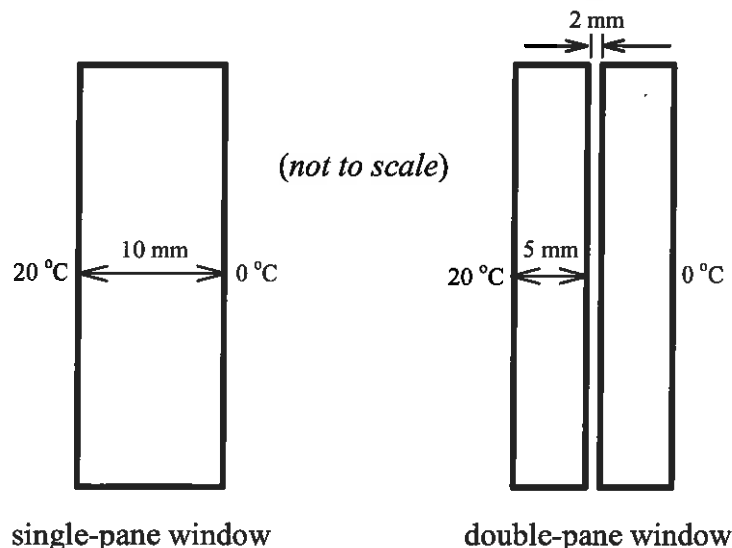
$$C(x, t) = \frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt}$$

- Show that $C(x, t)$ is a valid solution of Fick's second law of diffusion: $\partial C/\partial t = D\partial^2 C/\partial x^2$.
- Verify that the $C(x, t)$ solution of the diffusion equation is consistent with conservation of mass and therefore a constant number of diffusing molecules along the diffusion column:

$$N_0 = A \int_{-\infty}^{+\infty} C(x, t) dx$$

- Using $D = 1.55 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, $A = 0.500 \text{ cm}^2$, and $N_0 = 1.00 \times 10^{17}$, calculate the concentration gradient $\partial C/\partial x$ and the flux density $J_x = -D\partial C/\partial x$ at position $x = 0.15 \text{ m}$ at time $t = 1000 \text{ s}$.
- Photons travel at the speed of light, about 300,000 km per second. But it takes about 800,000 years for photons generated by thermonuclear fusion in the core of the sun to reach the surface of the sun (radius 696,000 km). Account for this apparent discrepancy in terms of photon diffusion.

7. a) The effective radius of a sucrose molecule is 0.49 nm. Use the Stokes-Einstein equation ($D = kT/6\pi r\eta$) to calculate the diffusion coefficient of sucrose in water at 60 °C (viscosity 0.000467 kg m⁻¹ s⁻¹).
- b) Estimate the time required for the root-mean-square displacement of diffusing sucrose molecules to reach 5.00 cm in water at 60 °C.
- c) Use the answer from b to explain why it is better to use stirring, not diffusion, to mix sugar into a cup of tea or coffee.
8. a) The thermal conductivity of argon at 300 K and 1.00 bar is 0.0177 J K⁻¹ m⁻¹ s⁻¹. Calculate the collision cross section and collision diameter of argon. This question illustrates that transport property measurements and kinetic theory can be used to estimate molecular sizes.
- b) At a given temperature and pressure, the thermal conductivity of krypton is roughly one half that of argon. Estimate the collision cross section and collision diameter of krypton.
9. Hydrogen gas is used as a high-performance coolant for the turbo-generators in electric power plants. Why not use air? Why not use helium? Explain.
10. a) Calculate the heat flux in kW through a **single-pane** plate-glass window of thickness 10 mm and area 16 m² if the inside temperature is 20 °C and the outside temperature is 0 °C. The thermal conductivity of window glass is 0.96 J K⁻¹ m⁻¹ s⁻¹.
- b) Recalculate the heat flux if the window in part a is replaced with a **double-pane** window consisting of a 2 mm thick layer of air (thermal conductivity 0.025 J K⁻¹ m⁻¹ s⁻¹) sandwiched between two 5 mm thick sheets of glass.
- c) Why are most windows double-pane, not single-pane?



Q1 Argon at 0.200 bar and 300 K in a 0.500 L container. Collision area $\sigma = 0.36 \text{ nm}^2$.

a) argon-argon collision rate per cubic meter:

upper case

$$z_{11} = \left(\frac{N_1}{V}\right)^2 \frac{\sigma}{\sqrt{2}} \sqrt{\frac{8KT}{\pi M_1}} = \left(\frac{P_1}{KT}\right)^2 \frac{\sigma}{\sqrt{2}} \sqrt{\frac{8RT}{\pi M_1}} = \frac{C_1^2}{\sqrt{2}} \langle v \rangle$$

$$= \left(\frac{0.200 \times 10^5 \text{ Pa}}{1.381 \times 10^{-23} \text{ J K}^{-1} (300 \text{ K})}\right)^2 \frac{0.36 \times 10^{-18} \text{ m}^2}{\sqrt{2}} \sqrt{\frac{(8) 8.314 \text{ J mol}^{-1} (300 \text{ K})}{\pi (0.03995 \frac{\text{kg}}{\text{mol}})}}$$

$$z_{11} = 2.365 \times 10^{33} \text{ m}^{-3} \text{ s}^{-1}$$

$$V = 0.500 \text{ L} = 0.000500 \text{ m}^3$$

collision rate in volume $V = z_{11} V$

$$= (2.365 \times 10^{33} \text{ m}^{-3} \text{ s}^{-1}) (0.000500 \text{ m}^3)$$

$$= 1.18 \times 10^{30} \text{ s}^{-1}$$

b) argon-wall collision rate per square meter:

$$z_c = \frac{P}{\sqrt{2\pi mKT}} = \frac{N_A P}{\sqrt{2\pi N_A m N_A K T}} = \frac{N_A P}{\sqrt{2\pi M R T}}$$

$$= \frac{(6.022 \times 10^{23} \text{ mol}^{-1}) (0.200 \times 10^5 \text{ Pa})}{\sqrt{2\pi (0.03995 \text{ kg mol}^{-1}) 300 \text{ K} (8.314 \text{ J K}^{-1} \text{ mol}^{-1})}}$$

all SI units

$$z_c = 4.813 \times 10^{26} \text{ m}^{-2} \text{ s}^{-1}$$

$$= 4.813 \times 10^{26} \text{ m}^{-2} \text{ s}^{-1} \left(10^{-4} \frac{\text{m}^2}{\text{cm}^2}\right) = 4.813 \times 10^{22} \text{ cm}^{-2} \text{ s}^{-1}$$

$$1 \text{ cm}^2 \leftrightarrow 0.0001 \text{ m}^2$$

per square cm

Q2 a) H atoms at 0.200 bar and $T = 1000 \text{ K}$
 Cl atoms at 0.300 bar

reduced mass $\mu = \frac{m_H m_{Cl}}{m_H + m_{Cl}}$

H-Cl collision rate

$$Z_{HCl} = C_H C_{Cl} \sigma \sqrt{\frac{8KT}{\pi\mu}}$$
 (units: $\text{m}^{-3}\text{s}^{-1}$)

reduced molar mass $\mu_M = \frac{M_H M_{Cl}}{M_H + M_{Cl}} = \frac{0.001008 (0.03545)}{0.001008 + 0.03545} \frac{\text{kg}}{\text{mol}}$

$\mu_M = 0.0009801 \text{ kg mol}^{-1}$

$\frac{N_H}{V} = C_H = \frac{P_H}{KT} = \frac{0.200 \times 10^5 \text{ Pa}}{1.381 \times 10^{-23} \text{ J K}^{-1} (1000 \text{ K})} = 1.448 \times 10^{24} \text{ m}^{-3}$

$\frac{N_{Cl}}{V} = C_{Cl} = \frac{P_{Cl}}{KT} = \frac{0.300}{0.200} C_H = 2.172 \times 10^{24} \text{ m}^{-3}$

H-Cl collision area $\sigma = \pi (r_H + r_{Cl})^2$

$\sigma = \pi [(0.10 + 0.17) \times 10^{-9} \text{ m}]^2 = 0.229 \times 10^{-18} \text{ m}^2$

collision rate $Z_{HCl} = \frac{P_H}{KT} \frac{P_{Cl}}{KT} \sigma \sqrt{\frac{8KT}{\pi\mu}} = C_H C_{Cl} \sigma \sqrt{\frac{8RT}{\pi\mu_M}}$
 $= (1.448 \times 10^{24}) (2.172 \times 10^{24}) (0.229 \times 10^{-18}) \sqrt{\frac{8(8.314)1000}{\pi 0.0009801}}$

$Z_{HCl} = 3.35 \times 10^{33} \text{ m}^{-3} \text{ s}^{-1}$

H-Cl collisions per second in a 0.500 L volume =

$Z_{HCl} V = 3.35 \times 10^{33} \text{ m}^{-3} \text{ s}^{-1} (0.500 \text{ L}) (10^{-3} \text{ m}^3 \text{ L}^{-1})$
 $= 1.68 \times 10^{30} \text{ s}^{-1}$



b) molecular reaction rate

$$C_H = \frac{N_H}{V}$$

$$C_{Cl} = \frac{N_{Cl}}{V}$$

$$\frac{dC_H}{dt} = \frac{dC_{Cl}}{dt} = -k_2 C_H C_{Cl}$$

molecular rate constant k_2 has units $\frac{m^3}{s}$

C_H and C_{Cl} have units $\frac{1}{m^3}$

maximum possible rate constant if every H-Cl collision produces HCl

$$\left(\frac{dC_H}{dt}\right)_{max} = \left(\frac{dC_{Cl}}{dt}\right)_{max} = -Z_{HCl} = -k_{2(max)} C_H C_{Cl}$$

$$= -\sigma \sqrt{\frac{8RT}{\pi \mu_M}} C_H C_{Cl}$$

$$k_{2(max)} = \sigma \sqrt{\frac{8RT}{\pi \mu_M}}$$

$$k_{2(max)} = 1.06 \times 10^{-15} m^3 s^{-1}$$

$$\left(\begin{array}{l} C_H = \frac{N_{Av}}{V} [H] (10^{-3} m^3 L^{-1}) \\ C_{Cl} = \frac{N_{Av}}{V} [Cl] (10^{-3} m^3 L^{-1}) \end{array} \right)$$

molar reaction rate

$$\frac{d[H]}{dt} = \frac{d[Cl]}{dt} = -k_{2M} [H][Cl]$$

molar rate constant k_{2M} has units $L mol^{-1} s^{-1}$

compare: $\frac{dC_H}{dt} = \frac{dC_{Cl}}{dt} = -k_2 C_H C_{Cl}$

$$\frac{d}{dt} \left(\frac{N_{Av}}{10^{-3} m^3 L^{-1}} [H] \right) = \frac{d}{dt} \left(\frac{N_{Av}}{10^{-3} m^3 L^{-1}} [Cl] \right) = -k_2 \frac{N_{Av}}{10^{-3} m^3 L^{-1}} [H] \frac{N_{Av}}{10^{-3} m^3 L^{-1}} [Cl]$$

$$\frac{d[H]}{dt} = \frac{d[Cl]}{dt} = - \left(\frac{k_2 N_{Av}}{10^{-3} m^3 L^{-1}} \right) [H][Cl]$$

$$k_{2M(max)} = \frac{N_{Av}}{10^{-3} m^3 L^{-1}} k_{2(max)} = \frac{(6.022 \times 10^{23} mol^{-1}) (1.06 \times 10^{-15} m^3 s^{-1})}{10^{-3} m^3 L^{-1}}$$

$$= 6.41 \times 10^{11} L mol^{-1} s^{-1}$$

Q3 $D = \frac{3\pi}{8} \frac{1}{2} \langle v \rangle \lambda$

average speed $\langle v \rangle = \sqrt{\frac{8RT}{\pi M}} \propto \frac{1}{\sqrt{M}}$

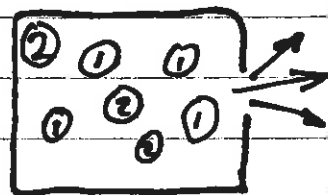
mean free path $\lambda = \frac{1}{\frac{N}{V} \sqrt{2} \sigma} = \frac{kT}{\sqrt{2} p \sigma} \propto \frac{1}{\sigma}$

at a given temperature and pressure, the diffusion coefficient is proportional to $\frac{1}{\sqrt{M}}$ and $\frac{1}{\sigma}$

helium atoms are both lighter and smaller than argon atoms, so $D_{He} > D_{Ar}$

Q4 rate of effusion through a hole of area A:

$$Z_c A = \frac{P}{\sqrt{2\pi m k T}}$$

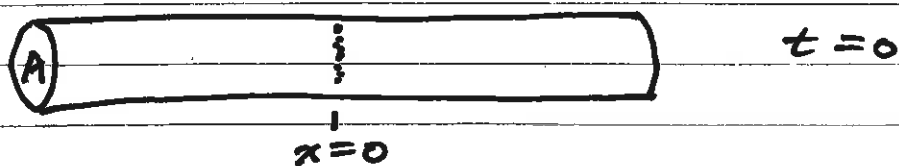


at a given temperature, the rate of effusion of a molecule is proportional to its partial pressure and inversely proportional to its mass

$$\frac{dN_i}{dt} \propto \frac{P_i}{\sqrt{m_i}} \propto \frac{N_i}{\sqrt{m_i}} \quad \frac{N_i}{V} = \frac{P_i}{kT}$$

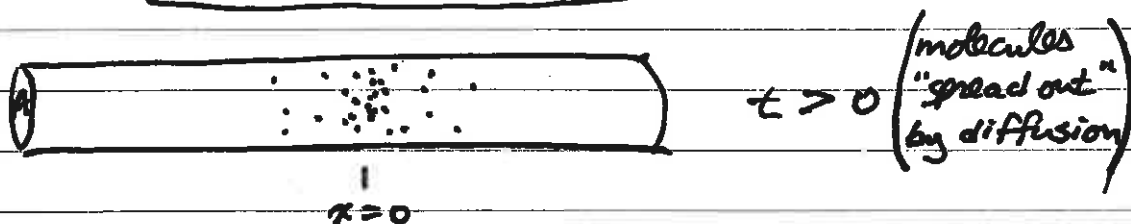
the proportion of lighter molecules in the effused gas will be higher than in the original gas mixture in the effusion cell

(Q5) a) N_0 molecules are initially located at $x=0$ in a tube of cross-sectional area A



After diffusion for time t , the concentration of molecules along the tube is

$$C(x,t) = \frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt}$$



Prove $\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$

Fick's Second Law

$$L.S. = \frac{\partial C}{\partial t} = \frac{\partial}{\partial t} \left[\frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt} \right]$$

$$= \frac{N_0}{2A\sqrt{\pi Dt}} \frac{x^2}{4Dt^2} e^{-x^2/4Dt} + \frac{N_0}{2A\sqrt{\pi D}} \left(-\frac{1}{2} t^{-3/2} \right) e^{-x^2/4Dt}$$

$$= \frac{N_0}{2A\sqrt{\pi Dt}} \left(\frac{x^2}{4Dt^2} - \frac{1}{2t} \right) e^{-x^2/4Dt}$$

$$= \left(\frac{x^2}{4Dt^2} - \frac{1}{2t} \right) C(x,t)$$

(5 a cont.)

$$\begin{aligned}RS &= D \frac{\partial}{\partial x} \frac{\partial C}{\partial x} = D \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} \frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt} \right) \\&= D \frac{N_0}{2A\sqrt{\pi Dt}} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} e^{-x^2/4Dt} \right) \\&= D \frac{N_0}{2A\sqrt{\pi Dt}} \frac{\partial}{\partial x} \left(\frac{-2x}{4Dt} e^{-x^2/4Dt} \right) \\&= D \frac{N_0}{2A\sqrt{\pi Dt}} \left(\frac{-2}{4Dt} e^{-x^2/4Dt} - \frac{2x}{4Dt} \left(\frac{-2x}{4Dt} \right) e^{-x^2/4Dt} \right) \\&= \frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt} \left(\frac{-2D}{4Dt} + \frac{4x^2 D}{16D^2 t^2} \right)\end{aligned}$$

$$RS = C(x,t) \left(\frac{-1}{2t} + \frac{x^2}{4Dt^2} \right)$$

$$RS = LS$$

in volume element $A dx$, there are $dN = C(x,t) A dx$ molecules

b) is mass conserved?

the number of diffusing molecules calculated from $C(x,t)$ is constant

total diffusing molecules at time $t = \int dN$

$$= \int_{-\infty}^{\infty} C(x,t) A dx = \int_{-\infty}^{\infty} \frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt} A dx$$

$$= \frac{N_0}{2\sqrt{\pi Dt}} \int_{-\infty}^{\infty} e^{-x^2/4Dt} dx = \frac{N_0}{2\sqrt{\pi Dt}} \int_{-\infty}^{\infty} e^{-ax^2} dx$$

$$= \frac{N_0}{2\sqrt{\pi Dt}} \sqrt{\frac{\pi}{a}} = \frac{N_0}{2\sqrt{\pi Dt}} \sqrt{\frac{\pi}{1/4Dt}} = N_0 \checkmark$$

$a = \frac{1}{4Dt}$
 $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$
(definite integral)

(5 cont.)

flux density

concentration gradient

c) $J_x = -D \left(\frac{\partial C}{\partial x} \right)$ Fick's First Law

concentration gradient $\frac{\partial C}{\partial x} = \frac{\partial}{\partial x} \left(\frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt} \right)$

$$\frac{\partial C}{\partial x} = \frac{N_0}{2A\sqrt{\pi Dt}} \frac{\partial}{\partial x} e^{-x^2/4Dt}$$

$$= \frac{N_0}{2A\sqrt{\pi Dt}} \frac{-2x}{4Dt} e^{-x^2/4Dt}$$

$$= -\frac{x}{2Dt} \frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt}$$

$$\frac{\partial C}{\partial x} = -\frac{x}{2Dt} C(x,t)$$

$$D = 1.55 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$$

$$A = 0.500 \text{ cm}^2 = 0.500 \times 10^{-4} \text{ m}^2$$

$$N_0 = 1.00 \times 10^{17}$$

at position $x = 0.150 \text{ m}$ at $t = 1000 \text{ s}$:

$$C(x,t) = \frac{N_0}{2A\sqrt{\pi Dt}} e^{-x^2/4Dt} = \frac{(1.00 \times 10^{17}) e^{-\frac{(0.150)^2}{4(1.55 \times 10^{-5})(1000)}}}{2(0.500 \times 10^{-4}) \sqrt{\pi (1.55 \times 10^{-5})(1000)}}$$

$$C(0.150 \text{ m}, 1000 \text{ s}) = (4.532 \times 10^{21} \text{ m}^{-3}) e^{-0.3629} = 3.152 \times 10^{21} \text{ m}^{-3}$$

$$\frac{\partial C}{\partial x} = -\frac{x}{2Dt} C(x,t) = -\frac{0.150 \text{ m}}{2(1.55 \times 10^{-5} \text{ m}^2 \text{ s}^{-1})(1000 \text{ s})} (3.152 \times 10^{21} \text{ m}^{-3})$$

$$\frac{\partial C}{\partial x} = -1.525 \times 10^{22} \text{ m}^{-4} \quad \text{concentration gradient (molecules per unit volume per meter)}$$

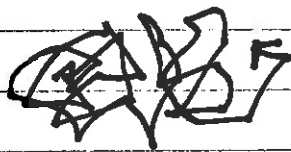
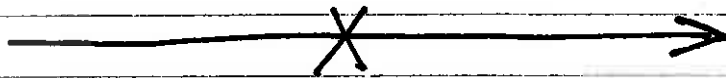
$$J_x = -D \frac{\partial C}{\partial x} = -\left(1.55 \times 10^{-5} \frac{\text{m}^2}{\text{s}} \right) (-1.525 \times 10^{22} \text{ m}^{-4})$$

$$J_x = +2.36 \times 10^{17} \text{ m}^{-2} \text{ s}^{-1} \quad \text{(molecules per unit area per unit time)}$$

Q6) The interior of the sun is a dense plasma consisting of electrons, protons, helium nuclei, etc.

Photons are constantly absorbed and re-emitted in different, random directions.

As a result, photons make little headway, even though their instantaneous velocity is very high



similar to the diffusion of molecules in a gas with average speeds of hundreds of meters per second but rms displacements $\sqrt{\langle x^2 \rangle} = \sqrt{2Dt}$ of a few mm per second (using $D \approx 10^{-5} \text{ m}^2 \text{ s}^{-1}$ for gases)

Q7) a) Sucrose ("sugar") diffusing in water at 60°C

Stokes-Einstein equation

$$D = \frac{kT}{6\pi r \eta}$$

$$T = 333 \text{ K}$$

$$r = 0.49 \times 10^{-9} \text{ m}$$

$$\text{viscosity } \eta = 0.000467 \text{ kg m}^{-1} \text{ s}^{-1}$$

$$D = \frac{(1.38 \times 10^{-23} \text{ J K}^{-1})(333 \text{ K})}{6\pi (0.49 \times 10^{-9} \text{ m})(0.000467 \text{ kg m}^{-1} \text{ s}^{-1})} = 1.07 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$$

b) rms displacement $\sqrt{\langle x^2 \rangle} = \sqrt{2Dt} = 0.0500 \text{ m}$ ↖ 5.00 cm

$$t = \frac{\langle x^2 \rangle}{2D} = \frac{(0.0500 \text{ m})^2}{2(1.07 \times 10^{-9} \text{ m}^2 \text{ s}^{-1})} = 1.17 \times 10^6 \text{ s} \quad (13.5 \text{ days})$$

c) \times dimension of a cup of coffee or tea \approx a few cm
diffusion is too slow (days) to mix the sugar,
stir instead

Q8 a) thermal conductivity of argon at 300 K, 1 bar

$1 \text{ W} = 1 \text{ J s}^{-1}$

$$\kappa = 0.0177 \text{ J K}^{-1} \text{ m}^{-1} \text{ s}^{-1}$$

$$= 0.0177 \text{ W K}^{-1} \text{ m}^{-1}$$

collision cross section?

from kinetic theory $\kappa = \frac{25\pi}{32} \sqrt{\frac{kT}{\pi m}} \frac{1}{\sigma} C_v$

for Ar (monatomic gas, no internal vibrations or rotations)

$$C_v = \frac{3}{2} k$$

solve for σ : $\sigma = \frac{25\pi}{32} \sqrt{\frac{kT}{\pi m}} \frac{1}{\kappa} C_v = \frac{25\pi}{32} \sqrt{\frac{RT}{\pi M}} \frac{1}{\kappa} \frac{3k}{2}$

$$\sigma = \frac{25\pi}{32} \sqrt{\frac{(8.314 \text{ J K}^{-1} \text{ mol}^{-1})(300 \text{ K})}{\pi \cdot 0.03995 \text{ kg mol}^{-1}}} \left(\frac{\text{K m s}}{0.0177 \text{ J}} \right) \frac{3}{2} \left(1.381 \times 10^{-23} \frac{\text{J}}{\text{K}} \right)$$

$$\sigma = 4.05 \times 10^{-19} \text{ m}^2 = 0.405 \text{ nm}^2$$

$$\sigma = \pi d_{\text{Ar}}^2 = \pi (r_{\text{Ar}} + r_{\text{Ar}})^2$$

$$d_{\text{Ar}} = \sqrt{\frac{\sigma}{\pi}} = 3.59 \times 10^{-10} \text{ m}$$

$$= 0.359 \text{ nm}$$

b) $\frac{\kappa_{\text{Kr}}}{\kappa_{\text{Ar}}} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{Kr}}} \frac{\sigma_{\text{Ar}}}{\sigma_{\text{Kr}}}}$

$$\frac{1}{2} = \sqrt{\frac{39.95}{83.80} \frac{0.405 \text{ nm}^2}{\sigma_{\text{Kr}}}}$$

$$\sigma_{\text{Kr}} = 0.560 \text{ nm}^2$$

$$d_{\text{Kr}} = 0.422 \text{ nm}$$

Q9 H₂ molecules are very light (~ 2 g mol⁻¹), even lighter than He (~ 4 g mol⁻¹) and relatively small (low collision area σ)

Notice the thermal conductivity is proportional to $\frac{1}{\sigma \sqrt{m}}$

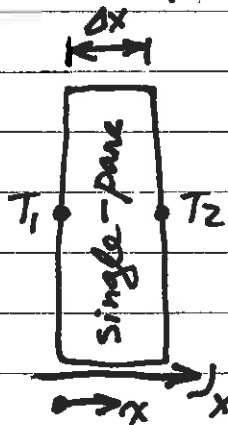
⇒ H₂ has the largest thermal conductivity of all gases
∴ a good coolant for rotating turbo-machinery

air — much lower thermal conductivity

helium — slightly lower thermal conductivity than H₂
but expensive

Q10 a) Calculate the heat flux through a sheet of glass, thickness 10 mm, area 16 m^2 , thermal conductivity $0.96 \text{ J K}^{-1} \text{ m}^{-1} \text{ s}^{-1}$, inside temperature 20°C , outside temperature 0°C
 (293 K) (273 K)
 T_1 T_2

$\Delta x = 10 \text{ mm} = 0.010 \text{ m}$



heat flux density $J_x = -\kappa \frac{\partial T}{\partial x} = -\kappa \frac{T_2 - T_1}{\Delta x}$

$J_x = -\left(\frac{0.96 \text{ J}}{\text{K m s}}\right) \frac{(273 - 293) \text{ K}}{0.010 \text{ m}}$

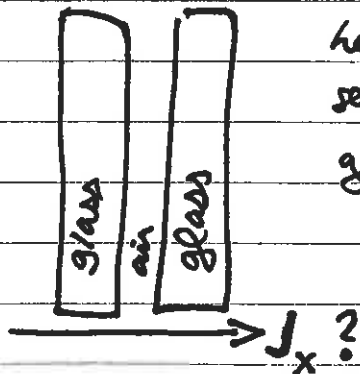
heat flow per unit area per second $J_x = 1920 \text{ J m}^{-2} \text{ s}^{-1}$

heat flow through area A per second $= J_x A = (1920 \text{ J m}^{-2} \text{ s}^{-1})(16 \text{ m}^2) = 30,720 \text{ W} = 30.72 \text{ kW}$

b) Calculate the heat flux through a double-pane window, same inside and outside temperature, same total thickness of glass (10 mm), but now a thin layer of air is in the path of the heat flow

The heat flux is constant:

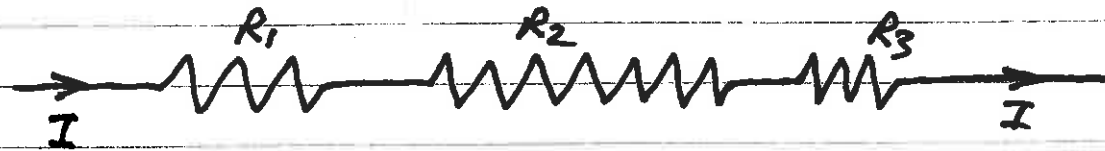
$J_x(\text{air}) = J_x(\text{glass})$



heat flows in series through glass, air, glass

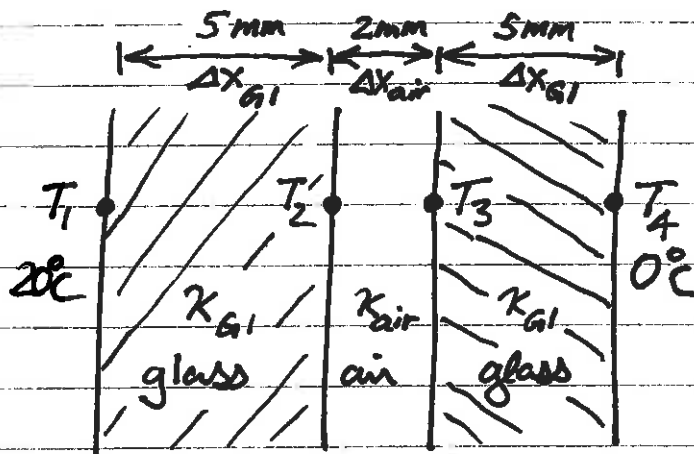
(10 5 cont.)

The flow of heat through two sheets of glass and the air-filled gap is analogous to the flow of electric current through three resistors in series:



Suggests treating the heat-flow problem in terms of resistance to heat flow, which is proportional to $\frac{1}{k}$ and Δx

$$J = -k \frac{\Delta T}{\Delta x} \implies \Delta T = -J \frac{1}{k} \Delta x$$



Note: The flux density of heat in the glass and in the air gap are identical.

$$k_{g1} = 0.96 \text{ J K}^{-1} \text{ m}^{-1} \text{ s}^{-1}$$

$$k_{air} = 0.025 \text{ J K}^{-1} \text{ m}^{-1} \text{ s}^{-1}$$

$$\left. \begin{aligned} T_2 - T_1 &= -\frac{J_x}{k_{g1}} \Delta x_{g1} \\ T_3 - T_2 &= -\frac{J_x}{k_{air}} \Delta x_{air} \\ T_4 - T_3 &= -\frac{J_x}{k_{g1}} \Delta x_{g1} \end{aligned} \right\} \text{add}$$

$$\Delta x_{g1} = 0.0050 \text{ m}$$

$$\Delta x_{air} = 0.0020 \text{ m}$$

$$T_4 - T_1 = -20 \text{ K}$$

$$\underline{\underline{T_4 - T_1 = -\left(\frac{\Delta x_{g1}}{k_{g1}} + \frac{\Delta x_{air}}{k_{air}} + \frac{\Delta x_{g1}}{k_{g1}}\right) J_x}}$$

(106 cont.)

$$\text{total } \Delta T = T_A - T_i = -20 \text{ K}$$
$$\text{total } \Delta x = \Delta x_{\text{gl}} + \Delta x_{\text{air}} + \Delta x_{\text{gl}} = 0.0120 \text{ m}$$

flux density of heat through the double-pane window:

$$J_x = - \left(\frac{\Delta x_{\text{gl}}}{k_{\text{gl}}} + \frac{\Delta x_{\text{air}}}{k_{\text{air}}} + \frac{\Delta x_{\text{gl}}}{k_{\text{gl}}} \right)^{-1} (T_A - T_i)$$

$$J_x = - \left(\frac{\Delta x_{\text{gl}} / \Delta x}{k_{\text{gl}}} + \frac{\Delta x_{\text{air}} / \Delta x}{k_{\text{air}}} + \frac{\Delta x_{\text{gl}} / \Delta x}{k_{\text{gl}}} \right)^{-1} \frac{\Delta T}{\Delta x}$$

$$= - \left(\frac{5 \text{ mm} / 12 \text{ mm}}{0.96} + \frac{2 \text{ mm} / 12 \text{ mm}}{0.025} + \frac{5 \text{ mm} / 12 \text{ mm}}{0.96 \frac{\text{J}}{\text{K m s}}} \right)^{-1} \frac{-20 \text{ K}}{0.0120 \text{ m}}$$

"effective" thermal conductivity

$$= - \left(0.133 \text{ J K}^{-1} \text{ m}^{-1} \text{ s}^{-1} \right) \frac{-20 \text{ K}}{0.0120 \text{ m}}$$

$$J_x = 221 \text{ J m}^{-2} \text{ s}^{-1} \quad \text{heat flux per square meter} \\ = \text{heat flux density}$$

for a window of area $A = 16 \text{ m}^2$

$$\text{heat flux} = J_x A = (221 \text{ J m}^{-2} \text{ s}^{-1})(16 \text{ m}^2) \\ = 3540 \text{ J s}^{-1} = 3.54 \text{ kW}$$

This question shows that the effective thermal conductivity for the flow of heat through a series of layers of different materials is:

$$k_{\text{eff}} = \left(\sum_i \frac{\Delta x_i / \Delta x}{k_i} \right)^{-1} \quad J_x = -k_{\text{eff}} \frac{\Delta T}{\Delta x}$$

$$\text{total thickness } \Delta x = \sum_i \Delta x_i$$

c) Switching from single-pane to double-pane construction reduces the heat flux through the window from 30.7 kW to 3.54 kW \Rightarrow lower heating cost! (also reduces air conditioning costs in hot weather)