Gradient geometry optimization leads us to a stationary point on PES gradE=0

It might be minimum, saddle point, inflection point, or maximum

How to find what the stationary point is (the nature of a stationary point)?

Like in calculus: get second derivatives of $E$ on nuclear coordinates
$\frac{\partial^{2} E}{\partial q_{i}^{2}}>0$
All second derivatives are positive- minimum on PES
$\frac{\partial^{2} E}{\partial q_{i}^{2}}=0$
All second derivatives are positive but one equals to zero-inflection point on PES
$\frac{\partial^{2} E}{\partial q_{i}{ }^{2}}<0$
All second derivatives are positive but one is negativesaddle point on PES- TRANSITION STATE!

If we know second derivatives of $E$ on $q_{i}$ (the force constants) we can calculate some physical properties. The most important are harmonic vibrational frequencies- gives us ideal IR spectrum

Classic harmonic oscillator
$F=-k x, k$ is a spring force constant

$$
v=1 / 2 \pi(k / \mu)^{1 / 2}
$$



Use similar model for quantum harmonic oscillator


Use Born-Oppenheimer approximation: allow nuclei to vibrate with frozen wave function!

So, if we have one negative force, $k$,
The square root will give as $(-1)^{1 / 2}=$, the imaginary unit. We'll get one imaginary frequency in the IR spectrum in G03 denoted as "-"

## Wave function. Molecular Orbitals as Linear Combination of Atomic Orbitals (MO LCAO)

Do we know exact AO for atoms?

Exact AO- tables of numbers
Approximate AO with a function
Approximate wave function with linear combination of exponential functions
Slater function

$$
\varphi_{A O}=N\left(r^{n^{*}-1} e^{-\zeta 1 r}+r^{n^{*}-1} e^{-\zeta 2 r}\right)
$$

AO as a linear combination of two Slater functions with 2 zeta variables Double-zeta basis set


Black -true wave function Blue- one Slater function Red- Double-zeta

Two Slater functions provide good accuracy but integrals with $\mathrm{e}^{-\mathrm{r}}$ are too complicated

Gauss functions (bell functions)

$$
\left.\varphi_{A O}=N_{n}(\alpha)\right)^{n-1} \exp \left(-a r^{2}\right)
$$



We shall need many Gauss functions to approximate wave function but integrals are easy to calculate, so overall it is faster

## Basis Sets in G03

All-electron basis sets
Minimal basis set: STO-3G (Slater type orbital approximated with 3 G-functions

> Pople's basis sets

Split-valence basis sets
3-21G, 6-31G, 6-311G

We can add additional sets of G-functions: polarization (d,p,f) and diffuse (denoted as +)

$$
6-31+G(d) \quad 6-311++G(2 d f, p)
$$

Effective core potential (ECP) basis sets. Core electrons are described with potential. Valence electrons- as above

## For heavy atoms (starting from $4^{\text {th }}$ period)

## Relativistic core potential and associated basis sets

LanL2DZ (LanL-relativistic effective core potential, DZ-basis set for valence electrons) and SDD (Stuttgart -Dresden ECP and D-basis set)

Both model core electrons by a function Treat valence electrons explicitly

Core can be taken from Dirac equation- relativistic calculations.
Simulate relativistic effects
Relativistic effects (mass and velocity are related)
Special theory of relativity: space-time symmetry
For H -like atom velocity of 1 s electron is

$$
\mathrm{v}=\mathrm{Zc} / 137 \quad \mathrm{Z} \text { is a nuclear charge }
$$

For heavy atoms, velocity of 1 s e approaching speed of light- relativistic corrections are significant
d- and f-subshells expand

With relativistic
effects, atoms are more chemically active


So, LanL2DZ and SDD describe these sub-shell effects

Another result of relativistic effects- spin-orbit coupling. Big challenge!

In general, for any mani-electron atom, inner subshells are less chemically important than valence electrons form out shells

Go to contracted basis sets, contracted Gauss type orbital, CGTO

$$
\text { CGTO }=\Sigma c_{i} \text { PGTO Primitive Gauss-type orbitals }
$$

Examples:
STO-3G

| C | 1 s | 2 s | $2 p_{x}$ | $2 p_{y}$ | $2 p_{z}$ | 5CGTO, 15PGTO <br> H |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 s |  |  |  |  |  |  |

3-21G

| C | 1 s | 2 s | $2 \mathrm{~s}^{\prime}$ | $2 \mathrm{p}_{\mathrm{x}}$ | $2 \mathrm{p}_{\mathrm{x}}^{\prime}$ | $2 \mathrm{p}_{\mathrm{y}}$ | $2 \mathrm{p}_{\mathrm{y}}^{\prime}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PGTO | 3 | 2 | 1 | 2 | 1 | 2 | $2 p_{\mathrm{z}}$ |
| $2 p_{\mathrm{z}}^{\prime}$ |  |  |  |  |  |  |  |

Overall, 15 PGTO and 9 CGTO

| H | 1s | $1 s^{\prime}$ | 2 |
| :--- | :--- | :---: | :--- |
| 2CGTO, 4 |  |  |  |
| PGTO | 3 | 1 |  |

$$
6-31 \mathrm{G}(\mathrm{~d})=6-31 \mathrm{G}^{*}
$$


15 CGTO, 6+16+6=28 PGTO

H

$$
\begin{array}{lcl}
\text { 1s } & 1 \mathrm{~s}^{\prime} & \text { 2CGTO, } 4 \text { PGTO } \\
3 & 1
\end{array}
$$

Computational time is proportional to $N^{4}$ where N is number of basis functions

| $\mathrm{C}_{6} \mathrm{H}_{6}$ | CGTO | PGTO |
| :--- | :---: | :---: |
| STO-3G | 36 | 108 |
| 3-21G | 66 | 108 |
| $6-31 G(d)$ | 102 | 192 |

## Basis idea of diffuse and polarization functions



Diffuse functions are important whenever loosely bound electrons are present: Anions, excited states. Also for cations- to describe long-range interactions

> Polarization functions

A basis set that incorporates functions of higher angular quantum number ( d for carbon, p for H ) is called a polarization basis set.
It provides for displacement of electron density away from the nuclear center
$s$ polarized by $p$
How to describe polar H-F bond?


So, add polarization function to H (p for s-orbital)

$$
\mathrm{p} \text { polarized by d }
$$



Now can describe nicely small bond angles
f-polarization functions allow to describe any volume of space around atomexpensive

