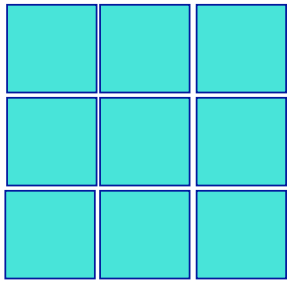
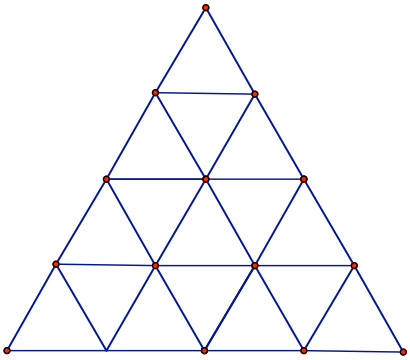
Math 423A Problem Set 1

1. Counting Squares

Consider the 3 by 3 grid given here. Can you count all the squares? Do you see that there are 14 squares in all? How many squares would you have in a 4 by 4 grid or a 5 by 5 grid? Can you find a general rule to determine the number of squares for a given square grid? What size would the grid need to be to have at least 10 000 squares?

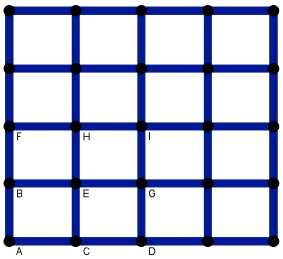


2. Extend the counting squares problem to explore equilateral triangles. How many triangles can you find in the shape below? Add more rows and see if you can find a general way to count triangles.



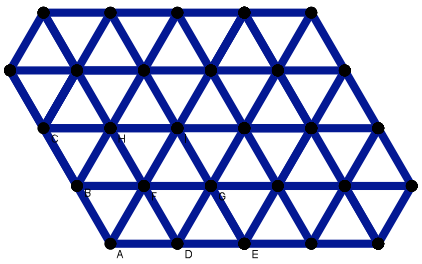
3. The Routes Problem
(Adapted from Points of Departure 1)

Start at A and travel along the lines moving only to the right (→) or up (↑). How many different ways can you get to each of the other points?

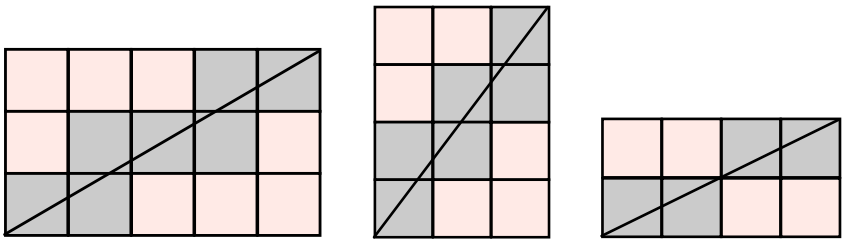


Extend beyond the labeled points. Can you spot some patterns? Can you make some generalisations about the pattern? Can you explain or prove them?

Suppose the grid changes and now you can move to the right and up diagonally in either direction, investigate the number of routes in this scenario.



4. Find out how many squares the diagonal of a rectangle passes through.



3 by 5 diagonal passes through 7 squares
4 by 3 diagonal passes through 6 squares
4 by 2 diagonal passes through 4 squares

Find a way to determine the number of squares that the diagonal passes through in any given rectangle. Explain your reasoning.

Extend to find the number of cubes passed through by the diagonal of a rectangular solid.

5. Consecutive Sums

Some numbers can be expressed as the sum of a string of consecutive positive numbers. Which numbers have this property? Explain how you know.

Examples:
5 = 2 + 3
9 = 2 + 3 + 4
12 = 3 + 4 + 5
18 = 3 + 4 + 5 + 6

6. The Locker Problem

Here is the famous locker problem: Imagine you are at a school that has 100 lockers, all shut. Suppose the first student goes along the row and opens every locker. The second student then goes along and shuts every other locker beginning with locker number 2. The third student changes the state of every third locker beginning with locker number 3. (If the locker is open the student shuts it, and if the locker is closed the student opens it.) The fourth student changes the state of every fourth locker beginning with number 4. Imagine that this continues until the 100 students have followed the pattern with the 100 lockers.

- At the end, which lockers will be open and which will be closed? Why?
- Which lockers have been switched the most often?
- How many lockers, and which ones, were touched exactly five times?

Explain your reasoning for each response.