Mental Math

Mental Computation

Grade Primary

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NOVA SCOTIA
Education
English Program Services

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Contents

Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

While there are many aspects to mental math, this booklet, *Mental Computation*, deals with fact learning, mental calculations, and computational estimation — mental math found in General Curriculum Outcome (GCO) B. Therefore, teachers must also remember to incorporate mental math strategies from the six other GCOs into their yearly plans for Mental Math, for example, measurement estimation, quantity estimation, patterns and spatial sense. For more information on these and other strategies see *Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rational for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost $1.90, can I buy them if I have $5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost $3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

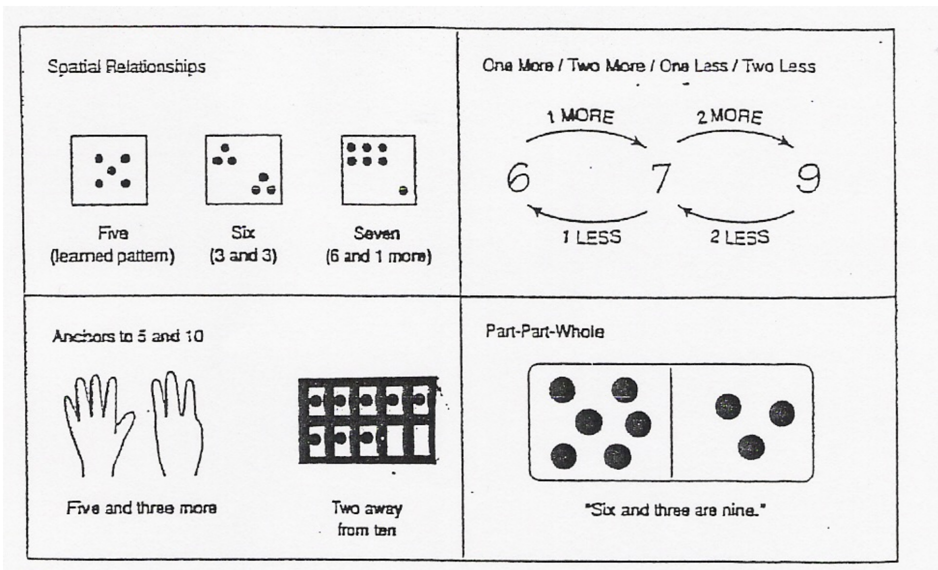
In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Grade Primary

While there is no mandated time allotted for mental math in grade primary, children need to develop some important concepts about number to prepare them for mental math learning in grade one. These concepts include:

- Counting
- Representing numbers
- Spatial relationships
- One more / Two more / One less / Two less
- Anchors to 5 and 10
- Part-part-whole



Throughout the year, students should be working toward developing these very important concepts using flash cards, games, die, ten frames, etc.

Every child learns differently and some concepts may take longer to develop than others. Students need to review previously learned concepts on a regular basis.

Counting

Description

Being able to count involves an understanding of the following principles:

- One number is said for each item in the group
- Counting begins with the number 1
- No item is counted twice
- The arrangement of objects is irrelevant
- The number in the set is the last number said

Activities

Observe students as they count:

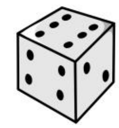
- do they touch each object as they count?
- do they set aside items/line them up as they count them?
- do they show confidence in their count or do they feel the need to check?
- do they check their counting in the same order as the first count or in a different order?
- need to start at the beginning to count additional objects?

Students will learn how to count forward from 1 and backward from 10. Some students may be able to count onward from a number i.e. 4 (5…6…7).

Representing Numbers

Description

Students need to be able to represent numbers. Students can practice making their numbers while performing meaningful counting or mathematical tasks. For example, students may be asked to roll a die and record the number of dots.



Activities

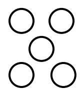
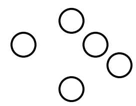
They may practice writing their phone number or record the number of counters when counting collections of objects.

		3			
1	2	3	4	5	6
1	2	3	4	5	6

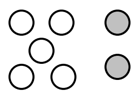
Spatial Relationships

Description

Students should recognize that there are many ways to arrange a set of objects, and that some arrangements are easier to recognize than others. Observe whether students are able to immediately say how many objects are displayed in familiar arrangements without doing a 1-to-1 count.

Ex: 5  **vs** 

For most numbers, there are common patterns (i.e. the ones found on dominoes and dice). Patterns for larger numbers can be made up of two or more easier patterns for smaller numbers.

7 

Activities

Learning Patterns with Dot Cards

To introduce patterns, provide each student with about 10 counters and a piece of construction paper as a mat. Hold up a dot card for about 3 seconds. Ask, "How many dots did you see? How did you see them? Make the pattern you saw using the counters on the mat". Spend some time discussing the configuration of the pattern and how many dots. Do this with a few new patterns each day

Dot Card/Plate Flash

Hold up a dot card for only 1 to 3 seconds. Ask, "How many? How did you see it?" Children like to see how quickly they can recognize and say how many dots. Include lots of easy patterns and a few with more dots as you build their confidence. Students can also flash the dot plates to each other as a workstation activity.

Dot Cards and Number Cards

Give each student a set of number cards (0-10). Hold up a dot card and have the students hold up the corresponding number card.

Dot Card Challenge

Two players each turn over a card from a stack of cards. The winner is the one with the larger total number and gets to take the cards (or whatever you wish to make as a rule). Children should be encouraged to determine who is the winner just by looking rather than counting.

Dot Card Differences

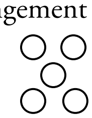
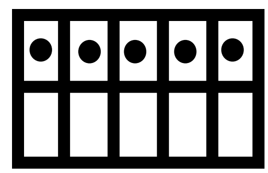
Students each have a pile of dot cards. There should also be a pile of about 50 counters. On each play, the players turn over their cards as usual. The player with the greater number of dots wins as many counters from the pile as the difference between the two cards. The players keep their cards. The game is over when the counter pile runs out. The player with the most counters wins the games.

One More / Two More / One Less / Two Less

Description

Students should learn how to count on and count back from a number without starting at the beginning. This would include counting on one more and two more and counting back one less and two less. In order to do this, students must understand the concepts of more and less as well as the counting sequence.

Ex: you could show students one of the following:

an arrangement of dots	number	ten frame
	5	

Ask, "what is one more?"
"what is two more?
"what is one less?"
"what is two less?"

You would only work on one of these at a time.

Encourage students not to start at the beginning but count on (or back) from the number being presented.

Activities

Make a One-More / One-Less Than Set (Two-More / Two-Less)

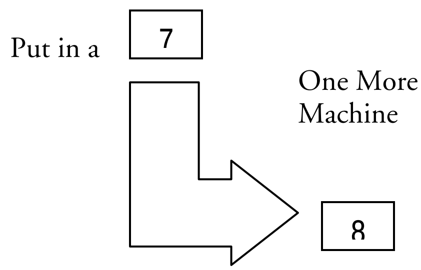
Hold up a dot card and have the students construct a set of counters that is one more than the set.
-…one less than the set
-…two more than the set
-…two less than the set

Dot Card: One-More / One-Less (Two-More / Two-Less)

Similar to the Dot Cards and Number Cards only ask the students to hold up the number card that is one more
-…hold up the number card that in one less
-…hold up the number card that is two more
-…hold up the number card that is two less

Number Machine

Draw a number machine on the board. It requires an input hopper and an output chute. Tell the children what the machine does; for example, "This is a magic one-more-than machine. It takes in a number up here and spits out a number that is one more."



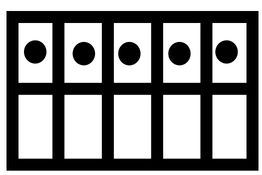
Dot Card Trains

Make a long row of dot cards from 0 up to 9, then go back again to 1, then up, and so on. Alternatively, begin with 0 or 1 and make a two-more/two-less train.

Ten-Frame flash (One-More/Two-More)

Flash a ten-frame card and ask the students to say one more or two more than the number of dots shown on the card.

Ex: **Show**

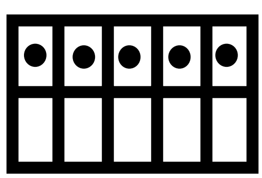


Students would say, "6"

Ten-Frame flash (One-Less/Two-Less)

Flash a ten-frame card and ask the students to say one more or two more than the number of dots shown on the card.

Ex: **Show**



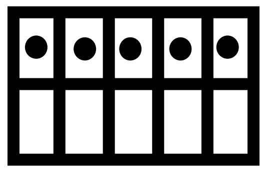
Students would say, "6"

Anchors to 5 and 10

Description

Ten plays an important role in our number system and since five and five make ten, it is important for students to develop and understand the relationships for the numbers 1 to 10 to the important anchors of 5 and 10.

Students will use the ten-frame to help develop and learn about these anchors.



Activities

Five-And game

The teachers calls out numbers between 5 and 10. The children respond "Five and ___" using the appropriate number. For example, if you say, "Eight!" the children respond, "Five and three."

Make-Ten game

Call out numbers between 0 and 10. The children respond by saying how many more are needed to make 10. This is most effective with numbers between 5 and 10.

Five Game

Hold up a ten frame and the children respond by stating the relationship to five. For example, show the ten frame with three dots. The children would respond, "Three is two less than five". Or holding up a ten frame with eight dots, the children would respond, "Eight is three more than five."

Ten Game

Hold up a ten frame and the children respond by stating the relationship to ten. For example, show the ten frame with three dots. The children would respond, "Three is seven less than ten". (these will always be less than statements except when ten is held up-"ten is the same as ten")

Part-Part-Whole

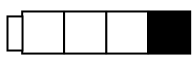
Description

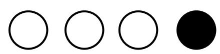
Children need to think about numbers as being made up of other numbers.

Ex: 6 is made up of 1 and 5, 2 and 4, 3 and 3, and 0 and 6.

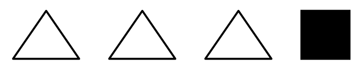
There is a clear connection between part-part-whole concepts and addition and subtraction. Children should be encouraged to make sets for numbers using a variety of materials.

Ex: cubes, counters, numbers, shapes Show the number **4**



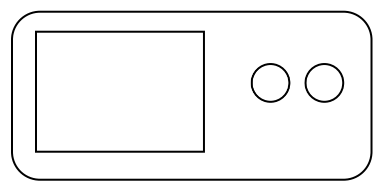


3 and 1



Related to part-part-whole is the missing-part concept where one of the two parts can be missing when the whole is known. This concept is necessary for understanding subtraction.

To show six in a missing-part activity, you might ask, "how many counters are hidden under the paper on the mat?" (the answer would be 4)



Part-Part-Whole Activities

Part-Part-Whole Mat

Each child has a part-part-whole mat and counters. Call out a number and the children place the counters on their mat in two parts to match the number. Ask for different configurations for the number. Eventually the children should be able to name the parts without the mats and counters.

Two-Part Dot Plates

Make a set of dot plates with solid and outline dots in various combinations to make numbers

Two-Part Dot Cards

Flash dot cards (with dots arranged in two groupings and have students say the two parts and the total)

Two out of Three

Make lists of three numbers, two of which total the whole that children are focusing on. Here is an example list for the number 5:

2-3-4
5-0-1
1-3-2
3-1-4
2-2-3
4-3-1

With the list on the board or overhead, children can take turns selecting the two numbers that make the whole. As with all problem-solving activities, children should be challenged to justify their answers. The same activity can be used in a worksheet format, but the real value lies in the discussion and justification.

Show with your Hands

Call out a number and have children show configurations with their fingers for that number

Missing-Part Activities

I Wish I Had

Hold up a ten frame or dot card/plate showing 7 or less. Say, "I wish I had seven." The children respond with the part that is needed to make 7. Counting on can be used to check. The game can focus on a single whole, or the "I Wish I Had" number can change each time.

Number Machine (see One more/Two more/One less/Two less activities)

Draw a funny looking machine on the board. It requires an input hopper and an output chute. Tell the children what the machine does; for example, "This is a magic parts of 8 machine. It takes in a number up here and spits out a number that is one more." If 3 goes in a parts-of-8 machine, a 5 comes out. Do not forget the discussion concerning how the students decided on the outcome.

Covered Parts

A set of counters equal to the target amount is counted out and the rest put aside. Put the counters in a cup or tub so the students can't see. Take some out of the cup and show the students. (This amount could be none, all, or any amount in between.) For example, if 6 is the whole and 4 are showing, the students should say, "Four and *two* is six". If there is hesitation or if the hidden part is unknown, the hidden part is immediately shown. The focus is on learning and thinking, not on testing and anxiety.