Mental Math
Mental Computation
Grade 4

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Contents

Introduction 1
Definitions 1
Rationale 1

The Implementation of Mental Computational Strategies 3
General Approach 3
Introducing a Strategy 3
Reinforcement 3
Assessment 3
Response Time 4

A. Addition — Fact Learning 5
Review Addition Facts and the Fact Learning Strategies 5

B. Addition — Mental Calculations 6
Quick Addition 6
Front End Addition 6
Finding Compatibles 7
Break Up and Bridge 8
Compensation 8
Make 10s, 100s, or 1000s 9

C. Subtraction — Fact Learning 11
Reviewing Subtraction Facts and the Fact Learning Strategies 11

D. Subtraction — Mental Calculations 11
Using Subtraction Facts for 10s, 100s, and 1000s 11
Quick Subtraction 12
Back Through 10/100 12
Up Through 10/100 13
Compensation 14
Balancing for a Constant Difference 14
Break Up and Bridge 15

E. Addition and Subtraction — Computational Estimation 16
Rounding 16
Front End 18
Adjusted Front End 18
Clustering of Near Compatibles 19

F. Multiplication — Fact Learning 20
Multiplication Fact Learning Strategies 20

G. Multiplication — Mental Calculations 23
Multiplication by 10 and 100 23

Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

While there are many aspects to mental math, this booklet, *Mental Computation*, deals with fact learning, mental calculations, and computational estimation — mental math found in General Curriculum Outcome (GCO) B. Therefore, teachers must also remember to incorporate mental math strategies from the six other GCOs into their yearly plans for Mental Math, for example, measurement estimation, quantity estimation, patterns and spatial sense. For more information on these and other strategies see *Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rational for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost $1.90, can I buy them if I have $5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost $3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

A. Addition— Fact Learning

Reviewing Addition Facts and Fact Learning Strategies

At the beginning of grade 4, it is important to ensure that students review the addition facts to 18 and the fact learning strategies. The addition facts should have been learned in Grade 2 and then applied to 10s, 100s, and 1000s in Grade 3. It would be a good idea to the relationship of the sums to the addition of two groups of base-10 blocks. For example, for 5 small cubes and 6 small cubes or 5 rods and 6 rods or 5 flats and 6 flats or 5 large cubes and 6 large cubes, the results will all be 11 blocks, be they 11 ones, 11 tens, 11 hundreds, or 11 thousands. The sums of 10s are a little more difficult than the sums of 100s and 1000s because when the answer is more than ten 10s, students have to translate the number. For example, for 70 + 80, 7 tens and 8 tens are 15 tens, or *one hundred fifty*.

Examples

The following are the Grade 2 fact strategies with examples and examples of the same facts applied to 10s, 100, and 1000s:

a) Doubles Facts (4 + 4, 40 + 40, 400 + 400, and 4000 + 4000)

b) Plus One (Next Number) Facts (5 + 1, 50 + 10, 500 + 100, 5000 + 1000)

c) 1-Apart (Near Double) Facts (3 + 4, 30 + 40, 300 + 400, 3000 + 4000)

d) Plus Two (Next Even/Odd) Facts (7 + 2, 70 + 20, 700 + 200, 7000 + 2000)

e) Plus Zero (No Change) Facts (8 + 0, 80 + 0, 800 + 0, 8000 + 0)

f) Make 10 Facts (9 + 6, 90 + 60, 900 + 600, 9000 + 6000
8 + 4, 80 + 40, 800 + 400, 8000 + 4000)

g) The Last 12 Facts with some possible strategies (may be others):

h) 2-Apart (Double Plus 2) Facts (5 + 3, 50 + 30, 500 + 300, 5000 + 3000)

i) Plus Three Facts (6 + 3, 60 + 30, 600 + 300, 6000 + 3000)

j) Make 10 (with a 7) Facts (7+ 4, 70 + 40, 700 + 400, 7000 + 4000)

Examples of Some Practice Items

40 + 40 =	7 000 + 9 000 =	100 + 200 =
90 + 90 =	8 000 + 6 000 =	4 000 + 2 000 =
50 + 50 =	3 000 + 5 000 =	
300 + 300 =	4 000 + 2 000 =	
7 000 + 7 000=	55 + 0 =	
2 000 + 2 000=	0 + 47 =	
70 + 80 =	376 + 0 =	
50 + 60 =	5 678 + 0 =	
7 000 + 8 000 =	0 + 9 098 =	
3 000 + 2 000 =	811 + 0 =	
40 + 60 =	70 + 20 =	
50 + 30 =	30 + 20 =	
700 + 500 =	60 + 20 =	
100 + 300 =	800 + 200 =	

B. Addition — Mental Calculations

Quick Addition (Extension)

This strategy is used when there are more than two combinations in the calculations, but no regrouping is needed. The calculations are presented visually instead of orally, and students will quickly record their answers on paper. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end. It is important to present examples of these addition questions in both horizontal and vertical formats.

Examples of Some Practice Items

a) Examples of Some Practice Items for Review of Numbers in the 10s and 100s:

71 + 12 =	34	56	25
63 + 33 =	+62	+31	+74
37 + 51 =			
291 + 703 =			
507 + 201 =	770 + 129 =		
623 + 234 =	534 + 435 =		

b) In Grade 4, this quick addition strategy is extended to sums involving thousands. Examples of Some Practice Items for Numbers in the 1000s:

6 621 + 2 100 =	300 + 2 078 =	5 200 + 3 700 =
1 4 52 + 8 200 =	7 600 + 2 064 =	6 245 + 1 712 =
4 423 + 1 200 =	6 334 + 2 200 =	4 678 + 3 211 =

Front-End Addition (Extension)

This strategy involves adding the highest place values and then adding the sums of the next place value(s).

Examples

a) i. For 37 + 26, think: 30 and 20 is 50, 7 and 6 is 13, and 50 plus 13 is 63.

ii. For 450 + 380, think: 400 and 300 is 700, 50 and 80 is 130, and 700 plus 130 is 830.

In Grade 4, this is front-end strategy is extended to sums involving thousands. Remember, however, the items for which this strategy would be applied should only involve two combinations. Also, recall that numbers in the thousands may appear with or without a space before the hundreds; however, tens of thousands must have a space.

Examples

a) i. 3300 + 2800, think: 3000 and 2000 is 5000, 300 and 800 is 1100, and + 5000 and 1100 is 6100.

ii. 2 070 + 1 080, think: 2 000 and 1 000 is 3 000, 70 and 80 is 150, and 3 000 and 150 is 3 150.

Examples of Some Practice Items

a) i. Examples of Some Practice Items for Numbers in the 10s:

34 + 18 =	53 + 29 =
15 + 66 =	74 + 19 =

ii. Examples of Some Practice Items for Numbers in the 100s:

190 + 430 =

340 + 220 =

470 + 360 =

607 + 304 =

b) Examples of Some Practice Items for Numbers in the 1000s (Grade 4):

4200 + 5300 =	6 100 + 2 800 =	3200 + 4500 =
7 700 + 1 100 =	5 200 + 3 400 =	4 700 + 2 400 =
6300 + 1800 =	7 800 + 2 100 =	10 300 + 4 400 =

Finding Compatibles (Extension)

This strategy for addition involves looking for pairs of numbers that combine easily to make a sum that will be easy to work with. In Grade 4, this should involve searching for pairs of numbers that add to 1000, another power of ten beyond 10 and 100 that were the focus in Grade 3. Some examples of common compatible numbers are: 1 and 9; 40 and 60; 300 and 700; and 75 and 25. (Compatible numbers are also referred to as *friendly* numbers or *nice* numbers in some professional resources.) You should be sure that students understand that in the numbers in addition can be combined in any order (associative property of addition).

Examples

For 3 + 8 + 7 + 6 + 2, think: 3 and 7 is 10, 8 and 2 is 10, so 10 and 10 and 6 is 26.

For 25 + 47 + 75, think: 25 and 75 is 100, so 100 plus 47 is 147.

For 400 + 720 + 600, think: 400 and 600 is 1000, and 1000 plus 720 is 1720.

Examples of Some Practice Items

Examples of Some Practice Items for Numbers in the 1s and 10s (Grade 3):
Add your own examples:

6 + 9 + 4 + 5 + 1 =	5	9
2 + 4 + 3 + 8 + 6 =	3	5
4 + 6 + 2 + 3 + 8 =	5	8
7 + 1 + 3 + 9 + 5 =	7	1
4 + 5 + 6 + 2 + 5 =	+4	+5
60 + 30 + 40 =	55	75
75 + 95 + 25 =	10	50

Examples of Some Practice Items for Numbers in the 100s (Grade 4):

300 + 437 + 700 =	310 + 700 + 300 =		
800 + 740 + 200 =	750 + 250 + 330 =	342	600
900 + 100 + 485 =	200 + 225 + 800 =	+500	+400

Break Up and Bridge (Extension)

This strategy involves starting with the first number in its entirety and adding the values in the place values of the second number one-at-a-time, starting with the largest. In Grade 4, this involves extending the practice items to numbers involving hundreds. Remember that the practice items should only include sums that involve two combinations.

Example

For 45 + 36, think: 45 and 30 (from the 36) is 75, and 75 plus 6 (the rest of the 36) is 81.

For 537 + 208, think: 537 and 200 is 737, and 737 plus 8 is 745.

In the introduction, you should model both numbers with base-10 blocks and model their addition by combining the blocks, starting with the largest, in the same way as you would combine the symbols.

Examples of Some Practice Items

Examples of Some Practice Items for Numbers in the 10s (Grade 3):

37 + 45 =	72 + 28 =	25 + 76 =
38 + 43 =	59 + 15 =	66 + 27 =

Examples of Some Practice Items for Numbers in the 100s (Grade 4):

325 + 220 =	301 + 435 =	747 + 150 =
439 + 250 =	506 + 270 =	645 + 110 =
142 + 202 =	370 + 327 =	310 + 518 =

Compensation (Extension)

This strategy involves changing one number in a sum to a nearby ten or hundred, carrying out the addition using that ten or hundred, and then adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of the change. In the last step it is helpful if they remind themselves that they added too much so they will have to take away that amount. Some students may have used this strategy when learning their facts involving 9s in Grade 2; for example, for 9 + 7, they may found 10 + 7 and then subtracted 1.

Examples

For 52 + 39, think: 52 plus 40 is 92, but I <u>added 1 too many</u> to take me to the next 10, so to compensate: **I subtract one from my answer**, 92, to get 91.

For 345 + 198, think: 345 + 200 is 545, but I added 2 too many; so I subtract 2 from 545 to get 543.

Examples of Some Practice Items

Examples of Some Practice Items for Numbers in the 10s (Grade3):
Add your own examples:

43 + 9 =	56 + 8 =	72 + 9 =
45 + 8 =	65 + 29 =	13 + 48 =
44 + 27 =	14 + 58 =	21 + 48 =

Examples of Some Practice Items for Numbers in the 100s (Grade 4):

255 + 49 =	371 + 18 =	125 + 49 =
504 + 199 =	326 + 298 =	412 + 499 =
826 + 99 =	304 + 399 =	526 + 799 =

Make 10s, 100s, or 1000s (Extension)

For single digit sums, if one addend is an 8 or 9, then a 2 or a 1 is taken from the other addend to turn the 8 or the 9 into a 10, and then the two *new* addends are easily combined. This "make-10" strategy would have been used in Grade 2 to learn the addition facts.

Example

a) i. For 9 + 6, think: 9 +1 (from the 6) is 10, and 10 + 5 (the other part of the 6) is 15.

Students should understand that this strategy centers on getting a more compatible addend, the 10. A common error occurs when students forget the other addend has changed as well. This strategy should be compared to the compensation strategy. As well, the "make 10" strategy can be extended to facts involving 7.

Example

ii. For 7 + 4, think: 7 and 3 (from the 4) is 10, and 10 + 1 (the other part of the 4) is 11.

In Grade 3, students would have applied this same strategy to sums involving single-digit numbers added to 2-digit numbers as a "make 10s" strategy.

Example

iii. For 58 + 6, think: 58 plus 2 (from the 6) is 60, and 60 plus 4 (the other part of 6) is 64.

Students are often excited when they notice that the ones digit in the answer is always the other part of the single-digit number.

In Grade 4, the strategy should be extended to "make 100s" and "make 1000s." Modelling some examples of the numbers with base-10 blocks, combining the blocks physically in the same way you would mentally, will help students understand the logic of the strategy.

Examples

b) For 350 + 59, think: 350 plus 50 (from the 59) is 400, and 400 plus 9 (the other part of 59) is 409.

For 7400 + 790, think: 7400 plus 600 (from the 790) is 8000, and 8000 plus 190 (the other part of 790) is 8190.

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 10s:
Add your own examples:

5 + 49 =

17 + 4 =

29 + 3 =

38 + 5 =

b) Examples of Some Practice Items for Numbers in the 100s:

680 + 78 =	490 + 18 =	170 + 40 =
570 + 41 =	450 + 62 =	630 + 73 =
560 + 89 =	870 + 57 =	780 + 67 =

Examples of Some Practice Items for Numbers in the 1000s:

2 800 + 460 =	5 900 + 660 =	1 700 + 870 =
8 900 + 230 =	3 500 + 590 =	2 200 + 910 =
3 600 + 522 =	4 700 + 470 =	6 300 + 855 =

C. Subtraction — Fact Learning

Reviewing Subtraction Facts and Fact Learning Strategies

At the beginning of Grade 4, it is important to ensure that students review the subtraction facts to 18 and review the fact learning strategies. All subtraction facts can be done by a "think addition" strategy, especially by students who know their addition facts very well.

D. Subtraction — Mental Calculations

Using Subtraction Facts for 10s, 100s, and 1000s (New)

This strategy applies to calculations involving the subtraction of two numbers in the tens, hundreds, and thousands with only one non-zero digit in each number. The strategy involves subtracting the single non-zero digits as if they were the single-digit subtraction facts. This strategy should be modeled with base-10 blocks so students understand that 7 blocks subtract 3 blocks will be 4 blocks whether the blocks are small cubes, rods, flats, or large cubes.

Example

For 80 – 30, think: 8 tens subtract 3 tens is 5 tens, or 50.

For 500 – 200, think: 5 hundreds subtract 2 hundreds is 3 hundreds, or 300.

For 9000 – 4000, think: 9 thousands subtract 4 thousands is 5 thousands, or 5000.

Examples of Some Practice Items

Examples of Some Practice Items for numbers in the 10s:

90 - 10 =	60 – 30 =	70 – 60 =
40 – 10 =	30 – 20 =	20 – 10 =
80 – 30 =	170 – 40 =	770 – 50 =

Examples of Some Practice Items for numbers in the 100s:

700 – 300 =	400 – 100 =	800 – 700 =
600 – 400 =	200 – 100 =	500 – 300 =
300 – 200 =	1400 – 100 =	1800 – 900 =

Examples of Some Practice Items for numbers in the 1000s:

2 000 – 1000 =	8 000 – 5 000 =
7 000 – 4 000 =	9 000 – 1 000 =
6 000 – 3 000 =	4 000 – 3 000 =
10 000 – 7 000 =	10 000 – 8 000 =

Quick Subtraction (New)

This pencil-and-paper strategy is used when there are more than two combinations in the calculations, but no regrouping is needed. The practice items are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads, starting at the front end. It is important to present these subtraction questions both horizontally and vertically.

Examples

For 86 - 23, simply record, starting at the front end, 63.

For 568 - 135, simply record, starting at the front end, 433.

Examples of Some Practice Items

Examples of Some Practice Items for numbers in the 10s:

38 – 25 =	76	85	48
27 – 15 =	-34	-31	-23
97 – 35 =			
78 – 46 =	45 – 30 =		
82 – 11 =	67 – 43 =		

Examples of Some Practice Items for numbers in the 100s:

745 – 23 =	624	846	537
947 – 35 =	-112	-324	-101
357 - 135 =			
845 – 542 =	704 – 50 2 =		639 – 628 =
452 – 311 =	809 – 408 =		8605 – 304 =

Back Through the 10/100(Extension)

This strategy extends one of the strategies students learned in Grade 3 for fact learning. This strategy involves subtracting a part of the subtrahend to get to the nearest tens or hundreds, and then subtracting the rest of the subtrahend. *This strategy is probably most effective when the subtrahend is not too great.*

Examples

For 15 – 8, think: 15 subtract 5 (one part of the 8) is 10 and 10 subtract 3 (the other part of the 8) is 7.

For 74 – 6, think: 74 subtract 4 (one part of the 6) is 70 and 70 subtract 2 (the other part of the 6) is 68.

For 530 – 70, think: 530 subtract 30 (one part of the 70) is 500 and 500 subtract 40 (the other part of the 70) is 460.

Examples of Some Practice Items

Examples of Some Practice Items for numbers in the 10s:
Add your own examples:

15 – 6 =	42 – 7 =	34 – 7 =
13 – 4 =	61 - 5 =	82 – 6 =
13 – 6 =	15 – 7 =	14 – 6 =
74 – 7 =	97 – 8 =	53 – 5 =

Examples of Some Practice Items for numbers in the 100s:

850 – 70 =	970 – 80 =	810 – 50 =
420 – 60 =	340 – 70 =	630 – 60 =
760 – 70 =	320 – 50 =	462 – 70 =

Up Through 10/100 (Extension)

This strategy is an extension of the "counting up through 10" strategy that students learned in Grade 3 to help learn the subtraction facts. This strategy involves counting the difference between the two numbers by starting with the smaller, keeping track of the *distance* to the nearest ten or hundred, and adding to this amount the rest of the *distance* to the greater number. *This strategy is most effective when the two numbers involved are quite close together*.

Examples

For 12 – 9, think: It is 1 from 9 to 10 and 2 from 10 to 12; therefore, the difference is 1 plus 2, or 3.

For 84 – 77, think: It is 3 from 77 to 80 and 4 from 80 to 84; therefore, the difference is 3 plus 4, or 7.

For 613 – 594, think: It is 6 from 594 to 600 and 13 from 600 to 613; therefore, the difference is 6 plus 13, or 19.

Examples of Some Practice Items

Examples of Some Practice Items for numbers in the 10s:

15 -8 =	14 – 9 =	16 – 9 =
11 -7 =	17 – 8 =	13 – 6 =
12 – 8 =	15 – 6 =	16 – 7 =
95 – 86 =	67 – 59 =	46 – 38 =
58 – 49 =	34 – 27 =	71 – 63 =
88 – 79 =	62 – 55 =	42 – 36 =

Examples of Some Practice Items for numbers in the 100s:

715 – 698 =	612 – 596 =	817 – 798 =
411 – 398 =	916 – 897 =	513 – 498 =
727 – 698 =	846 – 799 =	631 – 597 =

Compensation (New)

This strategy for subtraction involves changing the subtrahend to the nearest ten or hundred, carrying out the subtraction, and then adjusting the answer to compensate for the original change.

Examples

For 17 – 9, think: 17 – 10 = 7, but I subtracted 1 too many; so, I add 1 to the answer to compensate to get 8.

For 56 – 18, think: 56 – 20 = 36, but I subtracted 2 too many; so, I add 2 to the answer to get 38.

For 85 – 29, think: 85 – 30 + 1 = 56.

For 145 – 99, think: 145 – 100 is 45, but I subtracted 1 too many; so, I add 1 to 45 to get 46.

For 756 – 198, think: 756 – 200 + 2 = 558.

Examples of Some Practice Items

Examples of Some Practice Items for numbers in the 10s:
Add your own examples:

15 – 8 =	17 – 9 =	83 – 28 =
74 – 19 =	84 – 17 =	92 – 39 =
65 – 29 =	87 – 9 =	73 – 17 =

Examples of Some Practice Items for numbers in the 10s:

673 – 99 =	854 – 399 =	953 – 499 =
775 – 198 =	534 – 398 =	647 – 198 =
641 – 197 =	802 – 397 =	444 – 97 =
765 – 99 =	721 – 497 =	513 – 298 =

Balancing for a Constant Difference (New)

This strategy for subtraction involves adding or subtracting the same amount from both the subtrahend and the minuend to get to a ten or a hundred in order to make the subtraction easier. This strategy needs to be carefully introduced to convince students that this works because the two new numbers are the same distance apart as the original numbers. Examining possible numbers on a metre stick that are a fixed distance apart can help students with the logic of this strategy. Because both numbers change, many students may need to record at least the first changed number to keep track.

Examples

For 87 – 19, think: Add 1 to both numbers to get 88 – 20: so, 68 is the answer.

For 76 – 32, think: Subtract 2 from both numbers to get 74 –30; so, the answer is 44.

F or 345 – 198, think: Add 2 to both numbers to get 347 – 200; so, the answer is 147.

For 567 – 203, think: Subtract 3 from both numbers to get 564 -200; so, the answer is 364.

Examples of Some Practice Items

Examples of Some Practice Items for numbers in the 10s:

85 – 18 =	42 – 17 =	36 – 19 =
78 – 19 =	67 – 18 =	75 – 38 =
88 – 48 =	94 – 17 =	45 – 28 =
83 – 21 =	75 – 12 =	68 – 33 =
95 – 42 =	72 – 11 =	67 – 51 =
67 – 32 =	88 – 43 =	177 – 52 =

Examples of Some Practice Items for numbers in the 100s:
Add your own examples:

649 – 299 =	563 – 397 =	823 – 298 =
912 – 797 =	737 – 398 =	456 – 198=
631 – 499 =	811 – 597 =	628 – 298 =
971 – 696 =	486 – 201 =	829 – 503 =
659 – 204 =	382 – 202 =	293 – 102 =
736 – 402 =	564 – 303 =	577 – 101 =
948 – 301 =	437 – 103 =	819 – 504 =

Break Up and Bridge (New)

This strategy for subtraction involves starting with the first number (minuend) and subtracting, starting with the highest place value, the second number(subtrahend).

Examples

For 92 – 26, think: 92 subtract 20 (from the 26) is 72 and 72 subtract 6 is 66.

For 745 – 203, think: 745 subtract 200 (from the 203) is 545 and 545 minus 3 is 542.

Examples of Some Practice Items

Examples of Some Practice Items for Numbers in the 10s:

73 – 37 =	93 – 74 =	98 – 22 =
77 – 42=	74 – 15 =	77 – 15 =
95 – 27 =	85 – 46 =	67 – 42 =
52 – 33 =	86 – 54 =	156 – 47 =

Examples of Some Practice Items for Numbers in the 100s:

736 – 301 =	848 – 220 =	927 – 605 =
632 – 208 =	741 – 306 =	758 – 240 =
928 – 210 =	847 – 402 =	746 – 330 =
647 – 120 =	3580 – 130 =	9560 – 350 =

E. Addition and Subtraction — Computational Estimation

It is essential that estimation strategies are used by students before attempting pencil/paper or calculator computations to help them find "ball park" or reasonable answers.

When teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are: about, just about, between, a little more than, a little less than, close, close to and near.

Rounding (Extension)

A. Addition.

This strategy involves rounding each number to the highest, or the highest two, place values and adding the rounded numbers. Rounding to the highest place value would enable most students to keep track of the rounded numbers and do the calculation in their heads; however, rounding to two highest place values would probably require most students to record the rounded numbers before performing the calculation mentally.

When the digit 5 is involved in the rounding procedure for numbers in the 10s, 100s, and 1000s, the number can be rounded up or down. This may depend upon the effect the rounding will have in the overall calculation. For example, if both numbers to be added are about 5, 50, or 500, rounding one number *up* and one number *down* will minimize the effect the rounding will have in the estimation. Also, if both numbers are close to 5, 50, or 500, it may be better to round one up and one down.

Examples

For 378 + 230, think: 378 rounds to 400 and 230 rounds to 200; so, 400 plus 200 is 600.

For 45 + 65, think: since both numbers involve 5s, it would be best to round to 40 + 70 to get 110.

For 4 520 + 4 610, think: since both numbers are both close to 500, it would be best to round to 4 000 + 5 000 to get 9 000.

Examples of Some Practice Items

Examples of Some Practice Items for Rounding in Addition of Numbers in the 100s:

426 + 587 =	218 + 411 =	520 + 679 =
384 + 910 =	137 + 641 =	798 + 387 =
223 + 583 =	490 + 770 =	684 + 824 =
530 + 660 =	350 + 550 =	450 + 319 =
250 + 650 =	653 + 128 =	179 + 254 =

Examples of Some Practice Items for Rounding in Addition of Numbers in the 1000s:

5184 + 2 958 =	4 867 + 6 219 =	7 760 + 3 140 =
2 410 + 6 950 =	8 879 + 4 238 =	6 853 + 1 280 =
3 144 + 4 870 =	6 110 + 3 950 =	4 460 + 7 745 =
1 370 + 6 410 =	2 500 + 4 500 =	4 550 + 4 220 =

B. Subtraction

For subtraction, the process of estimation is similar to the addition, except for the situations where both numbers involve 5, 50, or 500 and where both numbers are close to 5, 50, or 500. For subtraction in these situations, both numbers should be rounded up because you are looking for the difference between the two numbers; so, you don't want to increase this difference by rounding one up and one down. This will require careful introduction for students to be convinced. (Help them make the connection to the Balancing for a Constant Difference strategy in mental math.)

Examples

To estimate 594 - 203, think: 594 rounds to 600 and 203 rounds to 200; so, 600 subtract 200 is 400.

To estimate 6237 – 2945, think: 6237 rounds to 6000 and 2945 rounds to 3000; so, 6 000 subtract 3000 is 3 000.

To estimate 5549 – 3487, think: both numbers are close to 500, so round both up; 6000 subtract 4000 is 2000.

Examples of Some Practice Items

Examples of Some Practice Items for Rounding in Subtraction of Numbers in the 100s: Add your own examples:

427 – 192 =	984 – 430 =	872 – 389 =
594 – 313 =	266 – 94 =	843 – 715 =
834 – 587 =	947 – 642 =	782 – 277 =

Examples of Some Practice Items for Rounding in Subtraction of Numbers in the 1000s: Add your own examples:

4768 – 3068 =	6892 – 1812 =	7368 – 4817 =
4807 – 1203 =	7856 – 1250 =	5029 – 4020 =
8876 – 3640 =	9989 – 4140 =	1754 – 999 =

Front End (Extension)

This strategy involves combining only the values in the highest place value to get a "ball- park" figure. Such estimates are adequate in many circumstances including getting an estimate before computations with technology in order to be alert to the reasonableness of the answers.

Examples

To estimate 243 + 354, think: 200 + 300 is 500.

To estimate 392 – 153, think: 300 subtract 100 is 200.

To estimate 437 + 541, think: 400 plus 500 is 900.

To estimate 534 – 254, think: 500 subtract 200 is 300.

To estimate 4 276 + 3 237, think: 4 000 plus 3 000 is 7 000.

To estimate 7896 – 2347, think: 7000 – 2000 is 5000.

Examples of Some Practice Items

Examples of Some Practice Items for Estimating Sums of Numbers in the 100s:

234 + 432 =	771 + 118 =	341 + 619 =
632 + 207 =	703 + 241 =	423 + 443 =
512 + 224 =	534 + 423 =	816 + 111 =

Examples of Some Practice Items for Estimating Differences of Numbers in the 100s:

327 – 142 =	928 – 741 =	804 – 537 =
639 – 426 =	718 – 338 =	248 – 109 =
431 – 206 =	743 – 519 =	823 – 240 =

Examples of Some Practice Items for Estimating Sums of Numbers in the 1000s:

1 324 + 8 265 =	5 719 + 4 389 =	4 096 + 3 227 =
7 261 + 2 008 =	2 467 + 5 106 =	4 275 + 2 105 =
6 125 + 2 412 =	5 489 + 3 246 =	3 321 + 6 410 =

Examples of Some Practice Items for Estimating Differences of Numbers in the 1000s:

6 237 – 2 945 =	5 475 – 3 128 =	8 289 – 1 443 =
9 153 – 2 611 =	4 308 – 1 489 =	8 452 – 5 134 =
9 496 – 5 008 =	9 240 – 3 170 =	7 189 – 2 364 =

Adjusted Front End (Extension)

This strategy begins by getting a Front End estimate and then adjusting that estimate to get a better, or closer, estimate by either (a) considering the second highest place values or (b) by clustering all the values in the other place values to "eyeball" whether there would be enough together to account for an adjustment.

Examples

a) To estimate 437 + 545, think: 400 plus 500 is 900, but this can be adjusted by thinking 30 and 40 is 70 which is closer to another 100; so, the adjusted estimate would be 900 + 100 = 1000.
or
To estimate 437 + 545, think: 400 plus 500 is 900, but this can be adjusted by considering that 37 and 45 would be close to another 100; so, the adjusted estimate would be 900 + 100 = 1000.

b) To estimate 3237 + 2125, think: 3000 plus 2000 is 5000, and 200 plus 100 is only 300, which is not close to another 1000 (or similarly "eyeballing" 237 plus 125 would result in no adjusting); so, the estimate is 5000.

c) To estimate 382 – 116, think: 300 subtract 100 is 200, and 80 – 10 is 70 that is close to another 100; so, the adjusted estimate is 300.
or
To estimate 382 – 116, think: 300 subtract 100 is 200, and "eyeballing" 82 – 16 suggests another 100 estimate; so, the adjusted estimate is 200 + 100 = 300.

d) To estimate 5674 – 2487, think: 5000 subtract 2000 is 3000, and 600 – 400 is 200 that is not close to another thousand; so, the estimate stays at 3000.

or
To estimate 5674 – 2487, think: 5000 subtract 2000 is 3000, and “eyeballing” 674 – 487 suggests there is not another thousand; so, the estimate stays at 3000.

Examples of Some Practice Items

Examples of Some Practice Items for Estimating Sums:

256 + 435 =	519 + 217 =	327 + 275 =
627 + 264 =	519 + 146 =	148 + 455 =
5423 + 2218 =	2518 + 1319 =	7155 + 5216 =

Examples of Some Practice Items for Estimating Differences:

645 – 290 =	720 – 593 =	834 – 299 =
935 – 494 =	468 – 215 =	937 – 612 =
7742 – 3014 =	4815 – 2709 =	2932 – 1223 =
9612 – 3424 =	5781 – 1139 =	4788 – 2225 =

Clustering of Near Compatibles (New)

When adding a list of numbers it is sometimes useful to look for two or three numbers that can be grouped to make 10 and 100 (compatible numbers). If there are numbers (near compatibles) that can be adjusted slightly to produce these compatibles, it will make finding an estimate easier.

Examples

For 44 + 33 + 62 + 71, think: 44 and 62 is almost 100, and 33 and 71 is almost 100; so, the estimate would be 100 + 100 = 200.

For 208 + 489 + 812 + 509, think: 208 and 812 is about 1000, and 489 and 509 is about 1000; so, the estimate is 1000 + 1000 = 2000.

For 612 – 289 + 397, think: 612 and 397 is about 1000, and 1000 subtract about 300 is 700.

Examples of Some Practice Items

32 + 62 + 71 + 41 =	76 + 81 + 22 + 24 =
51 + 21 + 53 + 82 =	11 + 71 + 92 + 33 =
33 + 67 + 72 =	67 – 8 - 2 + 21 =
44 + 38 + 62 =	52 – 3 – 7 + 10 =
73 – 11 – 22 + 1 =	153 – 31 - 22 + 1 =
476 – 74 + 27 - 33 =	239 – 43 + 54 - 62 =

F. Multiplication— Fact Learning

Multiplication Fact Learning Strategies

In grade 4, students are to learn the multiplication facts to achieve a 3-second response by the end of the year. This is done through learning a series of strategies, each of which addresses a cluster of facts. Each strategy is introduced, reinforced, and assessed before being integrated with previously learned strategies. It is important that students understand each strategy, so the introductions of the strategies are very important. As students master each set of the facts for each strategy, it is a good idea to have them record these learned facts on a multiplication chart so they may visually see their progress and know which facts they should be practicing. What follows is a suggested sequencing of strategies.

A. The Twos Facts (Doubles)

This strategy involves connecting the addition doubles to the related "two-times" multiplication facts. It is important to make sure students are aware of the equivalence of commutative pairs (2 × ? and ? × 2); for example, 2 × 7 is the double of 7 and that 7 × 2, while it means 7 groups of 2, has the same answer as 2 × 7. When students see 2 × 7 or 7 × 2, they should think: 7 and 7 are 14. Flash cards displaying the facts involving 2 and the times 2 function on the calculator are effective reinforcement tools to use when learning the multiplication doubles.

It is suggested that 2 × 0 and 0 × 2 be left until later when all the zeros facts are done.

B. The Nifty Nine Facts

The introduction of the facts involving 9s should concentrate on having students discover two patterns in the answers; namely, the tens' digit of the answer is one under the number of 9s involved, and the sum of the ones' digit and tens' digit of the answer is 9. For example, for 6 × 9 = 54, the tens' digit in the product is one less than the factor 6 (the number of 9s) and the sum of the two digits in the product is 5 + 4 or 9. Because multiplication is commutative, the same thinking would be applied to 9 × 6. Therefore, when asked for 3 × 9, think: the answer is in the 20s (the decade of the answer) and 2 and 7 add to 9; so, the answer is 27. You could help students master this strategy by scaffolding the thinking involved; that is, practice presenting the multiplication expressions and just asking for the decade of the answer; practice presenting the students with a digit from 1 to 8 and asking them the other digit that they would add to your digit to get 9; and conclude by presenting the multiplication expressions and asking for the answers and discussing the steps in the strategy.

Another strategy that some students may discover and/or use is a compensation strategy, where the computation is done using 10 instead of 9 and then adjusting the answer to compensate for using the 10 rather than the 9. For example, for 6 × 9, think: 6 groups of 10 is 60 but that is 6 too many (1 extra in each group) so 60 subtract 6 is 54. Because this strategy involves multiplication followed by subtraction, many students find it more difficult than the two-pattern strategy.

While 2 × 9 and 9 × 2 could be done by this strategy, these two nines facts were already handled by the twos facts. This nifty-nine strategy is probably most effective with numbers 3 to 9 times 9, leaving the 0s and 1s for later strategies.

C. The Fives Facts

If the students know how to read the various positions of the minute hand on an analog clock, it is easy to make the connection to the multiplication facts involving 5s. For example, if the minute hand is on the 6 and students know that means 30 minutes after the hour, then the connection to 6 × 5 = 30 can be made. This is why you may see the Five Facts referred to as the "clock facts." This would be the best strategy for students who know how to tell time on an analog clock.

Many students probably have been using a skip-counting-by-5 strategy; however, this strategy is difficult to apply in 3 seconds, or less, for all combinations, and often results in students' using fingers to keep track.

While most students have observed that the Five Facts have a 0 or a 5 as a ones' digit, some have also noticed other patterns; that is, the ones' digit is a 0 if the number of 5s involved is even and the ones' digit is 5 if the numbers of 5s involved is odd; the tens' digit of the answer is half the numbers of 5s involved, or half the number of 5s rounded up. For example, 8 × 5 ends in 0 because there are 8 fives and the tens' digit is 4 because 4 is half of 8; therefore, 8 × 5 is 40.

While this strategy applies to 2 × 5, 5 × 2, 5 × 9, and 9 × 5, these facts were also part of the twos facts, and nines facts. The fives facts involving zeros are probably best left for the zeros facts since the minute-hand approach has little meaning for 0.

D. The Ones Facts

While the ones facts are the "no change" facts, it is important that students understand why there is no change. Many students get these facts confused with the addition facts involving 1. To understand the ones facts, knowing what is happening when we multiply by one is important. For example 6 × 1 means *six groups of 1* or 1 + 1 + 1 + 1 + 1 + 1 and 1 × 6 means *one group of 6*. It is important to avoid teaching arbitrary rules such as "any number multiplied by one is that number". Students will come to this rule on their own given opportunities to develop understanding. Be sure to present questions visually and orally; for example, "4 groups of 1" and 4 × 1; and "1 group of 4" and 1 × 4.

While this strategy applies to 2 × 1, 1 × 2, 1 × 5, and 5 × 1, these facts have also been handled previously with the other strategies.

E. The Tricky Zeros Facts

As with the ones facts, students need to understand why these facts all result in zero because they are easily confused with the addition facts involving zero; thus, the zeros facts are often "tricky." To understand the zeros facts, students need to be reminded what is happening by making the connection to the meaning of the number sentence. For example: 6 × 0 means "six 0's or "six sets of nothing." This could be shown by drawing six boxes with nothing in each box. 0 × 6 means "zero sets of 6." This is much more difficult to conceptualize; however, if students are asked to draw two sets of 6, then one set of 6, and finally zero sets of 6, where they don't draw anything, they will realize why zero is the product. Similar to the previous strategy for teaching the ones facts, it is important not to teach a rule such as "any number multiplied by zero is zero". Students will come to this rule on their own, given opportunities to develop understanding.

F. The Threes Facts

The way to teach the threes facts is to develop a "double plus one more set" strategy. You could have students examine arrays with three rows. If they cover the third row, they easily see that they have a "double" in view; so, adding "one more set" to the double should make sense to them. For example, for 3 × 7, think: 2 sets of 7(double) plus one set of 7 or (7 × 2) + 7 = 14 + 7 = 21. This strategy uses the doubles facts that should be well known before this strategy is introduced; however, there will need to be a discussion and practice of quick addition strategies to add on the third set.

While this strategy can be applied to all facts involving 3, the emphasis should be on 3 × 3, 3 × 4, 4 × 3, 3 × 6, 6 × 3, 3 × 7, 7 × 3, 3 × 8, and 8 × 3, all of which have not been addressed by earlier strategies.

G. The Fours Facts

The way to teach the fours facts is to develop a "double-double" strategy. You could have students examine arrays with four rows. If they cover the bottom two rows, they easily see they have a "double" in view and another "double" covered; so, doubling twice should make sense. For example: for 4 × 7, think: 2 × 7(double) is 14 and 2 × 14 is 28. Discussion and practice of quick mental strategies for the doubles of 12, 14, 16 and 18 will be required for students to master their fours facts. (One efficient strategy is front-end whereby you double the ten, double the ones, and add these two results together. For example, for 2 × 16, think: 2 times 10 is 20, 2 times 6 is 12, so 20 and 12 is 32.)

While this strategy can be applied for all facts involving 4, the emphasis should be on 4 × 4, 4 × 6, 6 × 4, 4 × 7, 7 × 4, 4 × 8, and 8 × 4, all of which have not been addressed by earlier strategies.

H. The Last Nine Facts

After students have worked on the above seven strategies for learning the multiplication facts, there are only ***nine*** facts left to be learned. These include: 6 × 6; 6 × 7; 6 × 8; 7 × 7; 7 × 8; 8 × 8; 7 × 6; 8 × 7; and 8 × 6. At this point, the students themselves can probably suggest strategies that will help with quick recall of these facts. You should put each fact before them and ask for their suggestions.

Among the strategies suggested might be one that involves decomposition and the use of helping facts.

Examples

a) For 6 × 6, think: 5 sets of 6 is 30 plus 1 more set of 6 is 36.

b) For 6 × 7or 7 × 6, think: 5 sets of 6 is 30 plus 2 more sets of 6 is 12, so 30 plus 12 is 42.

c) For 6 × 8 or 8 × 6, think: 5 sets of 8 is 40 plus 1 more set of 8 is 48. Another strategy is to think: 3 sets of 8 is 24 and double 24 is 48.

d) For 7 × 7, think: 5 sets of 7 is 35, 2 sets of 7 is 14, so 35 and 14 is 49. (This is more difficult to do mentally than most of the others; however, many students seem to commit this one to memory quite quickly, perhaps because of the uniqueness of 49 as a product.)

e) For 7 × 8, think: 5 sets of 8 is 40, 2 sets of 8 is 16, so 40 plus 16 is 56. (Some students may notice that 56 uses the two digits 5 and 6 that are the two counting numbers before 7 and 8.)

f) For 8 × 8, think: 4 sets of 8 is 32, and 32 doubled is 64. (Some students may know this as the number of squares on a chess or chequer board.)

Multiplication — Mental Calculations

Multiplication by 10 and 100

This strategy involves keeping track of how the place values have changed. Introduce these products by considering base-10 block representations. For example, for 10 × 53, display 5 rods and 3 small cubes to represent 53, and think: 10 sets of 5 rods would be 50 rods, or 5 flats, and 10 sets of 3 small cubes would be 30 small cubes, or 3 rods; so, 5 flats and 3 rods represents 530. Through a few similar examples, it becomes clear that multiplying by 10 increases all the place values of a number by one place. For 10 × 67, think: the 6 tens will increase to 6 hundreds and the 7 ones will increase to 7 tens; therefore, the answer is 670.

Similarly, through modeling with base-10 blocks, it can be shown that multiplying by 100 increases all the place values of a number by two places. For 100 × 86, think: the 8 tens will increase to 8 thousands and the 6 ones will increase to 6 hundreds; therefore, the answer is 8 600.

While some students may see the pattern that one zero gets attached to the original number when multiplying by 10, and two zeros get attached when multiplying by 100, this is not the best way to introduce these products. Later, when students are working with decimals, such as 100 × 0.12, using the "two-place-value-change strategy" will be more meaningful than the "attaching-two-zeros strategy" and it will more likely produce a correct answer!

Examples of Some Practice Items

10 × 53 =	10 × 34 =	87 × 10 =
10 × 20 =	47 × 10 =	78 × 10 =
92 × 10 =	10 × 66 =	40 × 10 =
100 × 7 =	100 × 2 =	100 × 15 =
100 × 74 =	100 × 39 =	37 × 100 =
10 × 10 =	55 × 100 =	100 × 83 =
100 × 70 =	100 × 10 =	*40 × 100 =
5m = ___ cm	8m = ___cm	3m =___cm