Mental Math
Yearly Plan
Grade 7

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Contents

Introduction 1
Definitions 1
Rationale 1

The Implementation of Mental Computational Strategies 2
General Approach 2
Introducing a Strategy 2
Reinforcement 2
Assessment 2
Response Time 3

Mental Math: Yearly Plan — Grade 7 4
Number Sense 4
Fractions and Decimals 6
Decimals and Percent 9
Probability 12
Integers 13
Addition 13
Subtraction 14
Multiplication 16
Division 17
Geometry 18
Data Management 18
Patterns 19
Linear Equations and Relations 20

Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rational for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost $1.90, can I buy them if I have $5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost $3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

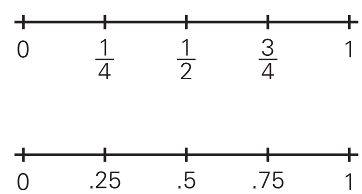
With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Mental Math: Grade 7 Yearly Plan

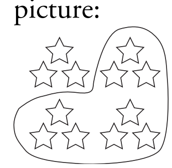
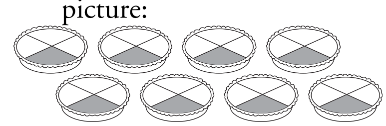
In this yearly plan for mental math in grade 7, an attempt has been made to align specific activities with the topic in grade 7. In some areas, the mental math content is too broad to be covered in the time frame allotted for a single chapter. While it is desirable to match this content to the unit being taught, it is quite acceptable to complete some mental math topics when doing subsequent chapters that do not have obvious mental math connections. For example practice with integer operations could continue into the data management and geometry chapters. Integers are so important in grade 7 that they should be interjected into the mental math component over the entire year once they have been taught.

	Skill	Example
Number Sense	Review multiplication and division facts through a) rearrangement, or commutative property/decomposition b) multiplying by multiples of 10 c) multiplication strategies such as doubles, double/double, double plus one, halve/double etc. (*Intent is to practice facts through previously learned strategies)*	a) $8 \times 7 \times 5 = 8 \times 5 \times 7$ (rearrangement or commutative property) $16 \times 25 = 4 \times 4 \times 25 = 4 \times 100$ b) $70 \times 80 = 7 \times 8 \times 10 \times 10$ $4\,200 \div 6 = (7 \times 600 \div 6)$ c) $12.5 \times 4 = 12.5 \times 2 \times 2$ (Double 12.5 and then double again) 16 x 25 8 x 50 (half 16 then double 25) $3 \times 15 = (2 \times 15) + (1 \times 15) = 30 + 15$
	Link exponents to fact strategies and properties for whole numbers. Use previously learned strategies such as - distributive strategy - associative property	a) $7^2 = 7 \times 7 = 49$ Use the above fact along with the distributive property to calculate: b) $7^3 = 7 \times 7 \times 7 = 49 \times 7 = 50 \times 7 - 7 = 350 - 7$ c) for 6^3, use distributive property $6 \times 6 \times 6 = 36 \times 6 = (30 \times 6) + (6 \times 6)$

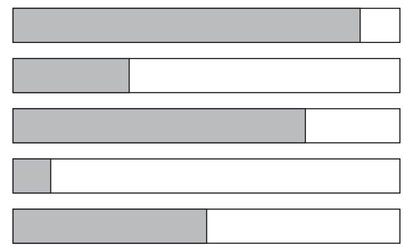
	Skill	Example
	- working by parts	d) $3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$ e) 2456 ÷ 8 = 2400 ÷ 8 + 56 ÷ 8 = 300 + 7 = 307
	Scientific notation: a) multiplying and dividing by powers of 10 b) dividing by 0.1 and multiplying by 10 give same result etc. c) practice converting between scientific and standard notations d) comparison of numbers in scientific notation or with powers of 10 computation	a) 24 000÷ 10 2.4 × 10 24 000 ÷ 100 0.24 × 100 24 000 ÷1000 0.024 × 1000 b) 0.024 ÷ 0.01 = 0.024 × 100 = 2.4 4.30 ÷ 0.001 = 4.30 × 1000= 4 300 c) Which exponent would you use to write these numbers in scientific notation? 87 000 = 8.7 x 10 $^{\bullet}$ 310 = 3.1 x 10 $^{\bullet}$ Write these numbers in standard form: 4×10^3 5.03×10^2 9.7×10^1 The correct scientific notation for the number 30100 is: 30.1×10^3 3.1×10^4 3.1×10^3 3.01×10^4 d) Which is larger: i) 5.07×10^4 or 2.4×10^8 ii) 2.3×10^5 or 234.7×10^2 iii) $670 \div 100$ or 6.7×100 iv) $\frac{88.9}{10}$ or $\frac{8.89}{0.01}$
	Apply the divisibility rules to working with factors and multiples	a) Which of these are prime? 2006, 2003, 2001, 1999 b) Is 1998 divisible by 4? 6? 9? c) Quick calculation -find the factors of 48 d) Quick calculation -find the first 5

	Skill	Example
	Apply the divisibility rules to help create multiples of numbers	multiples of 26 *(in c and d use pencils to record answers)* Fill in the missing digit(s) so that the number is a) divisible by 9: 3419__ b) divisible by 6: 7__158__ c) divisible by 6 and 9: 5601__ d) divisible by 5 and 6: 70__81__
Fractions and Decimals	The mental math material connected to this topic is extensive and consideration needs to be given as to what should be addressed during the fraction unit and what can be done at a later time.	
	Review mentally converting between improper fractions and mixed numbers	
	Teach benchmarks for fractions (0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1) and decimals (0, 0.25, 0.50, 0.75, 1) -treat fractions and decimals together	
	a) where they are located on a number line	a) Place these fractions in the appropriate place on the number line i. 63, $\frac{1}{17}$, $\frac{44}{51}$, $\frac{7}{16}$ ii. $\frac{1}{8}$, $\frac{1}{20}$, $\frac{1}{99}$, 0.001 iii. $\frac{33}{20}$, $\frac{6}{20}$, $\frac{17}{20}$, 2/20 iv. $\frac{7}{8}$, 0.42, 1 $\frac{1}{3}$, $\frac{13}{12}$, 0.15
	b) compare and order numbers with the benchmarks	b) Is $\frac{2}{5} < \frac{3}{4}$?

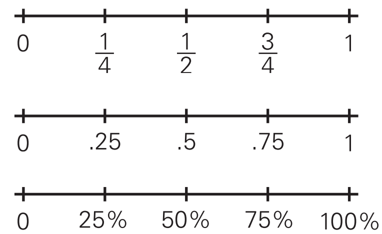
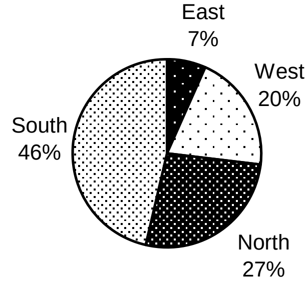
	Skill	Example
	c) Create fractions close to the bench marks	$0.51 > \frac{7}{8}$? $\frac{4}{3} < 1\frac{1}{2}$? Which benchmark is each of the following numbers closest to? 0.26, 0.81, 0.95, 0.00099 Which benchmark is each of the following numbers closest to? 0.51, 0.501, $\frac{1}{5}$, $\frac{4}{5}$, 0.9 c) complete the fraction so that it is close to the benchmark given: i. close to $\frac{1}{2}$: $\frac{\square}{11}$ ii. close to 1: $\frac{13}{\square}$ iii. close to 0.5: $\frac{25}{\square}$ iv. close to 0: $\frac{\square}{9}$
	Practice until students have automaticity of equivalence between certain fractions and decimals (halves, fourths, eights, tenths, fifths, thirds, ninths,) This can be revisited during the probability unit so the equivalencies are remembered and practiced. *Review mentally converting between improper fractions and mixed numbers*	Can use flashcards with fractions on one side and decimals on the other. If you have a class that works well together you can put fractions and decimals on separate cards and hand a 'class set' out. Go down the rows and as one student calls out their fraction or decimal the others have to listen and call out the equivalent if they have the card.
	- practice estimation using	Estimate: $\frac{2}{3} + \frac{1}{10}$

	Skill	Example
	benchmarks to add and subtract fractions and mixed numbers	$\frac{5}{8} \bullet \frac{10}{39}$ $4\frac{5}{6} - \frac{4}{5}$
	- is the sum or difference greater than, less than or equal to the closest benchmark	Is $\frac{1}{6} + \frac{1}{8} > \frac{1}{2}$? Is $\frac{2}{3} + \frac{3}{4} > 1.5$? Which sum or difference is larger? Estimate only? a) $\frac{3}{4}+\frac{4}{7}$ or $\frac{3}{8}+\frac{1}{10}$ b) $4\frac{5}{6}-\frac{2}{5}$ or $4\frac{5}{6}-\frac{3}{4}$
	a) Link multiplying a whole number by a fraction to division.	a) i. For $\frac{3}{4} \times 20$, **think** $\frac{1}{4}$ of 20 is the same as 20÷4 which is 5 so $\frac{3}{4} \times 20$ is 3 sets of 5 which is 15 ii. Write a fraction sentence for this picture: 
	b) Link multiplying a fraction by a whole number to visually accumulating sets	b) i. Write a fraction sentence for this picture:  ii. $6 \times \frac{1}{3}$ = 6 thirds = 2 wholes
	c) When the 2 separate visual pictures are firmly established,	c) i. $16 \times \frac{1}{8}$

	Skill	Example
	practice should consist of problems using both types *(The intent here is that students keep a firm connection between number sentences and visuals at this time.)*	ii. $\frac{4}{5} \times 10$ iii. $14 \times \frac{3}{7}$ iv. $\frac{2}{3} \times 9$
	Revisit the 4 properties associative, commutative, distributive, and identity a) mentally we want students to (1) recognize when a problem can be done mentally and (2) do the mental calculation b) create problems using whole numbers, decimals c) create problems that involve combinations of properties using rearrangement etc. Since students need much practice in this area, it is advisable to revisit this topic several times during the year, where appropriate.	Calculate: a) 1.33 + 8.25 + 6.75 b) 6 × 98 = 6 × (100 - 2) = (6 × 100) – (6 × 2) c) 4 × 2.25 c) 7 × 2.50 × 6 d) 25 × 2.08 × 4 e) 46 × 23 × 0 × 55 Judgment questions are found in the resource <u>Number Sense: Grades 6-8</u> (Dale Seymour Publications) pages 18 – 24
	Incorporate the "**Make 1, Make 10, etc**" strategy for decimals as well as properties stated above • do the 4 operations and incorporate other strategies	Practice can start with simple whole numbers, order of operations, and extend to decimals and use multiple strategies: a) 38 + 14 could be 38 + 2 + 12 b) 4 × 7 – 3 × 7 could be (4-3) x 7 c) 6 + 42 ÷7 d) $17 – 4^2$ e) 1.25 + 3.81 = 1.25 + 3.75 + 0.06 f) 4 × 0.26 = 4 × 0.25 + 4 × 0.01 g) 4- 1.98 could be 4.02- 2.00 or 4 – 2 + 0.02 h) 42 ÷ 0.07 = 4 200÷7
Decimals and Percent	a) This is an extension of the percents work done earlier. b) Establish benchmarks for percents: 0%, 25%, 50%, 75 %, and 100 %.	a) Use flashcards with fractions on one side and percents on the other. A suggested progression is to work with halves, fourths, tenths, and fifths on the first day and eighths, thirds and ninths on the next day. Mixed practice continues until automaticity is achieved. b) State the % benchmark closest to each of these:

	Skill	Example
		$\frac{11}{20}$, $\frac{1}{6}$, 0.98, $\frac{5}{8}$, $\frac{7}{15}$, $\frac{1}{11}$, 0.2
	a) Estimate % from a visual b) Convert easy fractions and decimals to a percent. c) Introduce "friendly fractions" in estimation of percents.	a) Estimate what % is shaded.  b) $\frac{4}{25}$ = __%, $\frac{7}{20}$ = __%, $\frac{1}{50}$ = __%, 0.16 = __%, 0.165 = __%, $\frac{25}{75}$ = __%, $\frac{20}{40}$ = __%, $\frac{6}{8}$ = __% c) Estimate the percentage: i. $\frac{23}{61} \rightarrow \frac{20}{60} = \frac{1}{3} = 33\frac{1}{3}\%$ ii. $\frac{141}{352} \rightarrow \frac{140}{350} = \frac{14}{35} = \frac{2}{5} = 40\%$ iii. $\frac{18234}{909000} \rightarrow \frac{18000}{900000} = \frac{18}{900}$ $= \frac{2}{100} = 2\%$
	a) Practice making judgments when given a problem as to whether to use the 1% method, convert the % to a fraction or convert the % to a decimal to solve the problem. b) Part to Whole problems using 1% method c) Estimation	a) Find i. 25 % of 80 (think $\frac{1}{4} \times 80$) ii. 23 % of 40 (think 0.23×40) iii. 6 % of 400 $\rightarrow$ 1 % of 400 = 4 , so 6 % of 400= $6 \times 4 = 24$ b) Find ☐ if 20% of ☐ is 8. c) 39% of 78 $\rightarrow$ 40 % of 80
	Mental math activities on sales tax will now become estimation activities instead of calculation activities. Students can	

	Skill	Example
	use the idea that 14 % is close to 15 % (10 + 5) to help them estimate a) Estimate sales tax mentally. 14% is close to 15% (10 %+ 5%) b) Create problems that can be done mentally using strategies learned previously. c) Estimate for discounts: -Applications to percents should be used when ever possible.	 a) 14% of $60 = think 14% is close to 15% so 10% x $60 + 5% x 60 b) Create a problem where you would convert the percent to i. a fraction. ii. a decimal iii. Use the 1% method c) i. Estimate the sale price for: 33 % discount of $39.99 → ii. Bring in a sales flyer and give the original price and the sale. Ask students to estimate the sale price of items from the flyer. As a motivator have the students pick an item from the flyer. Most flyers give the sale price so this will be easy to correct!
	Mixed practice using three kinds of percent problems: Work on judgment as well as solutions. This is an opportunity to review the Work by Parts, Halve / Double strategies. a) Find percentage b) Find the percent of a number c) Find the number when given the part (Use proportional reasoning) Look for the *Proportional Reasoning* resource in your school. There are lots of exercises at the front of the book to use to practice proportional reasoning with percents.	a) 6 is what % of 20? b) i. Work by Parts: 35% discount on $90.00 = (30 % of 90) + (5 % of 90) ii. Halve/double: $13\frac{1}{2}$ % of 200 = 26% of 100 c) If 15% of □ = 12, find □ Think 5 % of □ = 12÷3 = 4 so 10 % of □ = 8 and 100 % of □ is 80

Probability	There are opportunities to build on mental math content from previous work.	
	a)Reinforce equivalency among common fractions, decimals, and percent. b) Develop automaticity for conversions as well as associations with Never, Seldom, About half of the time, Often, and Always.	 a) Place on a number line: 25%, 0.49, $\frac{3}{4}$, 33% 90%, 0.10, 0.55, $\frac{2}{3}$, $\frac{4}{9}$ b) Use the descriptors "Never", Seldom", "About Half the Time", "Often" and "Always" to describe the following probabilities: $\frac{3}{4}$ 0.10 60% 88% 5% 0.45 $\frac{5}{9}$
	Finding a fraction of a whole number Estimation: a) Friendly fractions b) Interpreting graphs	In a survey of 30 members of a class, 12 answered Yes, and 16 No. What is the probability that the next person surveyed will say Yes? a) How many in a school of 700 would you expect to say No ? b) If the circle graph below describes a city population of 24 000, estimate how many people live in the northern part? 

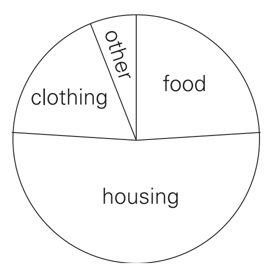
	Find theoretical probability mentally for common situations	What is the probability of – getting a 4 on a single roll of a die? – getting a prime number on a single roll of a die? – drawing a queen from a complete deck of cards – drawing a face card from a complete deck of cards?
Integers	Integers is a new topic for grade 7 and requires substantial teaching before students are able to practice working with the integers mentally. This reinforcement takes time and can be extended through the geometry work utilizing the beginning 5 – 7 minutes of class time.	
	Much time should be spent on the Zero principle. Students should realize that all integers can be expressed as a sum in many ways.	a) Using 2 color counters, make zero in three different ways. b) Give a context for each way. c) Can you make zero with __ counters? d) Can you make ___ with __ counters etc.? e) Write the integer that is: i. 5 larger than -2 ii. 4 less than -6
Addition	Much teaching needs to happen before the Mental Math can start. It should be tied to context. Begin with integers between -20 and +20 in size. Have students visualize the counters or relate to a context to help them work mentally. Come at the operations of addition and subtraction in as many ways as possible to help the students feel comfortable using the integers. This is also solidifying the conceptual knowledge of the operation as well as practicing the thinking behind the procedural knowledge.	Calculate: a) i. (+3) + (+4) ii. (+3) + (-4) iii. (-13) + (-4) iv. (+3) + (-3) b) Determine if a positive or a negative value has been added to give the sum in each: i. • 2 + □ = • 10 ii. 5 + □ = 8 iii. 7 + □ = 0 iv. □ • 8 = •16 c) Determine if the sum of the following integers would be positive or negative. <u>Do Not</u> give the sum: i. •8, 10

		ii. •12, •14 iii. 15, 20 iv. •4, •6, 13 v. 24, •25, •12 d) Give two integers, with different signs, that have a sum of : (Keep the integer values between 10 and -10) i. –2 (example: 4+-6 or –5+7) ii. –4 iii. 0 iv. 5 v. 7 vi. •1
Subtraction	This is the most difficult operation for students to internalize as their individual schemas for addition and subtraction are being rebuilt. Practice in the beginning should focus on context: - give a context and have students give the sentence - give a sentence and have students create a story. As the students become comfortable, longer number sentences with both addition and	a) Write an integer sentence to describe the person's net worth. i. John had 10 points in a card game, but lost 15 points in the next hand. ii. Peggy owed her mother $3.00, but received $8.00 for walking her aunt's dog. iii. Barbara's net worth was $6.00 until Donald forgave a debt of $5.00 that she had owed him. b) Create a story to fit each number sentence: i. (-3) - (+4) = (-7) ii. (-8) - (-10) = (+2) c) (+3) + (+4) – (-8) +(+7) (-4) + (-5) – (-4) + (-8)

	subtraction can be introduced. Remind students to continue to look for the zero principle in these longer addition/ subtraction sentences. Using integers, reinforce work with subtraction and operation sense.	d) Determine if a positive or a negative value is missing in each sentence: i. 4 − □ = −4 ii. □ − 5 = −2 e) Determine if the difference of the following integers would be positive or negative. Do Not give the difference: i. −4 − 8 = □ ii. 10 − 12 = □ iii. −4 − −6 = □ f) Give two integers that have a difference of: (Keep the integer values between 10 and -10) i. −1 ii. 3 iii. 4 iv. −5
	When these operations are firmly established, the strategies of − Front Ending,	g) i. (−194) + (−476) = ii. −500 + (−160) + (−10)
	− Finding Compatibles (or those integers that are close to opposites) and	h) i. (−124) + (+125) + (−476) = ii. (−124) + (−476) + +123) = iii. −600 + (+125 = −475
	− Compensation can be extended to integer sentences.	i) i. −580 − (−92) → −580 − (−100) = −480 ii. −480 + (−8) = −488

Multiplication	Early teaching should focus on giving meaning to multiplication expressions: Some contexts: a) (+3) × (-4) = -12: *Add 3 sets of -4 or borrow $4 for 3 days in a row*) b) (-3) × (+4)= -12: *Remove 3 sets of $4 from your net worth* c) (-3) × (- 4) = +12: *Remove 3 sets of -4*; *3 debts of $4. are forgiven.* *d)* (+3) × (+4) = +12: *Add 3 sets of $4. to your net worth.* Practice should be kept simple and in context. Have students explain the "sign patterns" they see as they work through problems as these sign patterns are key to division.	
	Fit context to number sentence	a) Calculate and be able to tell a story for each expression: i. (+3) × (-9) ii. (-5) × (-4) iii. (+3) × (+10) iv. (-8) × (+4) v. 0 × (-3)
	As students internalize sign patterns, strategies used for multiplying whole numbers can be extended to integers – Associative Property: – Halve and Double: – Distributive Law: – Compatible factors	b) i. (+3) × (+15) × (−2) = (+3) × (-30) ii. (-8) × (+35) = (−4) × (+70) iii. (-8) × (+56) = (−8) × [(+50 +6)] = (-8) × (+50) + (−8) × (6) iv. (+12) × (+25) × (-2) x(-4) = (-4) × (+25) x(-2) × (+12)

Division	In the elementary grades, students have learned that every multiplication sentence has 2 related division sentences. Students can examine the sign patterns “discovered” in multiplication of integers and through the writing of the related division sentences, extend these patterns to division. As with multiplication, early practice should be with smaller numbers until the sign patterns are automatic. Contexts should be asked for where possible. As students internalize sign patterns, strategies used for dividing whole numbers can be extended to integers See Guide 7-30 Balance before dividing (Multiply or divide both dividend and divisor by the same number): Work by parts:	a) Write two related division sentences for: i. (+3) × (+9) ii. (+3) × (-9) iii. (-3) × (+9) iv. (-3) × (-9) b) Write one related division sentence and one related multiplication sentence for: i. (+81) ÷ (-9) ii. (+32) ÷ (+4) iii. (-42) ÷ (-7) iv. (-54) ÷ (+9) v. 0 ÷ (-4) c) i. (-125) ÷ (+5) = (-250) ÷ (+10) ii. (-90) ÷ (-15) = (-30) ÷ (-5) iii. (-16) ÷ (+0.25) = (-64) ÷ (+1) d) i. (-1232) ÷ (-4) = (-1200) ÷ (-4) +(-32)÷ (-4) ii. (-128) ÷ (+8) = [(-80) + (+40)] ÷8
	Order of Operations with Integers: Create problems combining the 4 operations using smaller numbers at first and then moving to larger numbers. Include problems with brackets and exponents as well as some that can be done with strategies such as compatible numbers and front-end.	Calculate a) (+3) + (+5) x (-2) b) 20 – (-30) ÷ (+5) c) $(-2)^3$ d) $(-2)^4$ e) (-48) ÷ [(+32) ÷ (+4)] f) (-152) +(-248) – (-48) g) 12.5% of (-80)

Geometry	There are opportunities to sharpen subtraction skills when working with finding the sizes of angles. Subtract from the left Much of the Mental Math time in this unit could be spent on practice with integer skills. Students can practice creating angles close to 45 degrees, 90 degrees, and 180 degrees. This can be alternated throughout the week with integer skills or rational number skills.	a) i. 180 – 46 = 180 – 40 − 6 = 140 − 6 ii. 360 – 128 = 360 − 100 − 20 − 8 b) Put angles on the overhead, one at a time and have the students estimate the size of the angle. They should try to come within 5 degrees of the angle. c) Using only a straight edge and your pencil create the following angle measurements: i. 50 ii. 100 iii. 170
Data Management	Teachers can still use mental math time to reinforce integer operations.	
	Estimation activities that arise from interpreting data displays	a) Given a circle graph representing a total income of $50 000, use the sectors to estimate what percentage and what amount is spent on; Housing, Food, Clothing and Other.  b) Given a bar graph showing Mr. Jones's sales for the past six months, estimate his mean and median sales.

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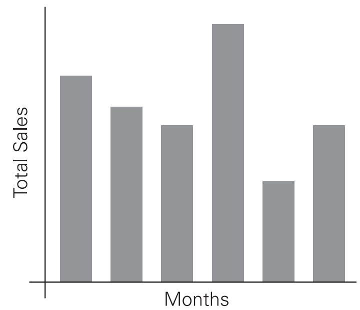
<td>
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<td>Compensation technique for finding mean. This gives a valuable application of adding and dividing integers</td>
<td>a) Find the mean of these grades:
85, 76, 71, 72, 86
· Choose 80 as a convenient central value, and then mentally compute the total of positive and negative differences of the central value from the mean. Divide this total by 5 (the number of integers given) and add to 80.
· +5 + (-4) + (-9) + (-8) + (+6) = -10
· Average difference: 10 ÷ (5) = -2
· Mean = 80 +(-2) = 78
b) Find the mean of this set:
46, 57, 49, 60, 48, 46</td>
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<td>Patterns</td>
<td>At the beginning of the unit, mental math should consist of practice using positive and negative numbers and order of operations, including brackets and exponents. This prepares the student for later work, such as evaluating algebraic expressions and solving single variable equations.

After the concept of the variable is firmly established, students should be given experiences where they can evaluate simple expressions and equations mentally.</td>
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<td>Have students extend number sequences using the 4 operations, exponents etc, visual patterns, tables</td>
<td>Give the next 3 terms in each sequence
a) −2, −4, −6, −8…
b) −5, −3, −1, …
c) −2, 4, −8, 16…
d) 100, 94, 88, …
e) −81, −27, −9…
f) 1, 4, 9, 16…
g) 1, 8, 27, 81…</td>
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<td>Have students examine tables to mentally determine the next term, a missing term or the nth term.</td>
<td>Determine the missing terms in these tables
<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>10</td><td>20</td><td></td><td>n</td></tr>
<tr><td>2</td><td>5</td><td>8</td><td></td><td></td><td></td><td>74</td><td></td></tr>
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<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>10</td><td>20</td><td></td><td>n</td></tr>
<tr><td>-4</td><td>-2</td><td>0</td><td>2</td><td></td><td></td><td>50</td><td></td></tr>
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<td>Mentally evaluating expressions</td>
<td>Evaluate each expression for
× = -2, ½, 0.2
a) $-6x$
b) $2x + 5$
c) x^2
d) $15 - x$</td>
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<td>Mentally be able to combine like terms and recognize the parallels to working with integers.</td>
<td>Simplify:
a) $3x - 2y + 4x - y$
b) $2a - 5 - 3a + 10$
c) $a - 2a - 3b + 6 - 4a + 5b - 4$</td>
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<td>Linear Equations and Relations</td>
<td>Work from the unit on Patterns can continue into linear relations and equations</td>
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<td>Once students have extensively practiced solving equations on paper, they can be introduced to some that can be done mentally and asked to orally explain the steps for solution.</td>
<td>a) $x + 5 = 7$
b) $z - 4 = -1$
c) $3w = -9$
d) $m \div 4 = -5$
e) $2q - 1 = 5$</td>
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<td>(9.5)</td>
<td>Have students mentally calculate rates such as: Better buy, beats/min, km/hr, $/hr – include conversions</td>
<td>a) Which is the better buy? One dozen peaches for $ 3.00 or 4 peaches for 90 cents.
b) John ran 35 metres in 15 seconds. How far could he run in 1 minute at the same speed?
c) If Kay earned $36.00 in 2.5 hours, how much would she earn in 5 hours? 10 hours? 1 hour?</td>
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