Section 3: Assessment and Evaluation

Introduction

Assessment and evaluation are essential to student success in mathematics. The purpose of assessment is manifold: Assessment yields rich data to evaluate student learning, the effectiveness of teaching, and the achievement of the prescribed curriculum outcomes. However, assessment without evaluation is insufficient, as the collection and reporting of data alone are not entirely useful unless the quality of the data is evaluated in relation to the outcomes. To this end, teachers use rubrics, criteria, marking keys, and other objective guides to evaluate the work of their students.

Assessment

Assessment is the process of collecting information about student learning (for example, through observation, portfolios, pencil-and-paper tests, performance). Assessment is the gathering of pertinent information.

Evaluation

Evaluation follows assessment by using the information gathered to determine a student's strengths, needs, and progress in meeting the learning outcomes. Evaluation is the process of making judgments or decisions based on the information collected in assessment.

Assessments *for* Learning

Assessments are designed with a purpose. Some assessments are designed by teachers as assessments "for" learning. The purpose of these assessments is, in part, to assist students in their progress towards the achievement of prescribed curriculum outcomes. In such assessments, the tasks used by teachers should inform students about what kinds of mathematical knowledge and performances are important.

As well, assessments for learning help teachers to know where their students are on the learning continuum, track each student's progress, and plan what "next steps" are required for student success. Following assessments for learning, teachers help students toward the achievement of a mathematics outcome by providing them with further opportunities to learn. In this way, such assessments take a developmental perspective and track students' growth through the year.

Assessment *as* Learning

Some assessments for learning are designed specifically to encourage student involvement and provide students with a continuous flow of information concerning their achievement. When students become involved in the process of assessment, it becomes assessment “as” learning. Assessment techniques such as conversation, interviews, interactive journals, and self-assessment help students to articulate their ideas and understandings and to identify where they might need more assistance. Such techniques also provide students with insight into their thinking processes and their understandings. This kind of assessment is used not only to allow students to check on their progress, but to advance their understandings, to encourage them to take risks, to allow them to make mistakes, and to enhance their learnings. This kind of assessment also helps students to monitor and evaluate their own learning, to take responsibility for their own record keeping, and to reflect on how they learn.

Teachers should keep in mind that such assessment practices may be unfamiliar to students at first, and that the emphasis on their being actively involved and thinking for themselves will be a challenge for some students. Such practices, however, enable teachers and students, together, to form a plan that ensures students are clear about what they have to do to achieve particular learning outcomes.

Assessments *of* Learning

Assessments “of” learning provide an overview of a student’s achievement in relation to the outcomes documented in the Atlantic Canada mathematics curriculum that form the basis for the student’s learning requirements. When an assessment of learning achieves its purpose, it provides information to the teacher for the grading of student work in relation to the outcomes.

Final assessments of learning should be administered after the student has had the fullest opportunity to learn the intended outcomes in the mathematics program. Assessments of learning check for a student’s achievement against the outcomes. It should be noted that any assessment for learning that reveals whether a student has met the intended outcome can also be considered assessment of learning, and the evaluation of that assessment may be used to report on the student’s achievement of the outcome.

Assessments “as,” “for,” and “of” learning are what teachers do in a balanced classroom assessment process.

Alignment

Assessments serve teaching and learning best when teachers integrate them closely with the ongoing instructional/learning process, when assessments are planned in advance, and when both formative and summative assessments are used appropriately. The nature of the assessments used by the teacher must be appropriate to and aligned with curriculum, so that students’ progress is measured by what is taught and what is expected. When learning is the focus, curriculum and assessment become opposite sides of the same coin, each serving the other in the interest of student learning and achievement. Assessments, therefore, should inform classroom decisions and motivate students by maximizing their confidence in themselves as learners. For this reason, teachers need to be prepared to understand the fundamental concepts of assessments and evaluation.

Choosing and using the right kinds of assessments are critical, and teachers need to be aware of the strengths and weaknesses of their assessment choices. As well, employing a variety of appropriate assessments improves the reliability of their evaluation and can help to improve both teaching and learning.

Assessment as and for learning are the foundation of classroom assessment activities leading to assessment of learning.

Planning Process: Assessment and Instruction

The Classroom Assessment Process

The following steps describe what the classroom assessment process might look like.

1.The teacher needs to have a clear understanding of the outcomes that are to be achieved and the multiple ways that students can be involved in the learning process. The teacher must

- be able to describe what the student needs to achieve
- collect and create samples that model what the learning looks like
- decide what kinds of evidence the student can produce to show that he or she has achieved the outcome(s) (i.e., design or select assessment tasks that allow for multiple approaches)
- select or create learning activities that will ensure student success

2. To introduce the learning, the teacher

- discusses the outcomes and what is to be achieved
- shows samples and discusses what the product of the learning should look like
- plans with students by setting criteria for success and developing time lines
- activates prior knowledge
- provides mini-lessons if required to teach/review prerequisite skills

3. After assigning the learning activity, the teacher

- provides feedback and reminds students to monitor their own learning during the activity
  - Feedback to any student should be about the particular qualities of the work, with advice on how to improve it, and should avoid comparisons with other students.
  - The feedback should have three elements: recognition of the desired performance evidence about the student's current understanding; and some understanding of a way to close the gap between the first two.
- encourages students to reflect on the learning activity and revisit the criteria

- The discourse in the classroom is imperative—the dialogue should be thoughtful, reflective, and focussed to evoke and explore understanding.
  - All students should have an opportunity to think and express their ideas.
  - The teacher must ask questions (see pages 111–125 for more discussion on "questioning").
  - The questions must require thought.
  - The wait time must be long enough for thinking to take place.
- uses classroom assessments to build student confidence
  - Tests given in class and exercises given for homework are important means for providing feedback.
  - It is better to have frequent short tests than infrequent long ones.
  - New learning should first be tested within about a week.
  - Quality test items, relevance to the outcomes, and clarity to the students are very important.
  - Good questions are worth sharing. Collaboration, between and among teachers, is encouraged.
  - Feedback from tests must be more than just marks.
  - Quality feedback from tests has been shown to improve learning when it gives each student specific guidance on strengths and weaknesses.
  - Instruction should be continuously adjusted, based on classroom assessments.
- encourages students to self-assess, review criteria, and set new goals to help them take responsibility for their own learning
  - Students should be taught self-assessment so that they can understand the purposes of their learning and understand what they need to do to improve. (See pages 129–130 for examples of self-assessment sheets.)
  - Students can reflect in their learning journals. (See page 129–130 for examples of prompts to promote writing in journals and logs.)

4. The teacher uses cumulative assessment.
   - Eventually, there is a time when students must be able to demonstrate what they have learned, what they understand, and how well they have achieved the outcomes.
   - Assessment should reflect the outcomes and should focus on the students' understanding, as well as their procedural skills. (See page 110 for examples of the kinds of questions students should be answering.)

Assessing Students' Understanding

What does it mean to assess students' understanding?

Students should be asked to provide evidence that they can

- identify and generate examples and non-examples of concepts in any representation (concrete, context, verbal, pictorial, and symbolic)
- translate from one representation of a concept to another

- recognize various meanings and interpretations of concepts
- identify properties and common misconceptions
- connect, compare, and contrast with other concepts
- apply concepts in new/novel/complex situations

What does it mean to assess students' procedural skills?

- Students should be asked to provide evidence that they can
- recognize when a procedure is appropriate
- give reasons for steps in a procedure
- reliably and efficiently execute procedures
- verify results of procedures analytically or by using models
- recognize correct and incorrect procedures

What follows is a stem problem and 5 questions that address the 5 bullets above. This addresses outcome 10+ C1 and 10D8, 10D14, 10E7 and 10E9

Marla is going to clean the outside of her bedroom window which is on the second floor of her house, 7.5 m above the ground. She has a 9 m ladder and positions it against the house so that it just reaches the bottom of the window. How far from the house on level ground is the foot of the ladder?

1. How would you find the answer to this problem?

2. Bobby's solution to this problem starts like this:

- $9^2 - 7.5^2 = x^2$.

Why did Bobby do this?

3. Solve the problem.

4. The following is Mary's solution. Has she made any errors or omissions? Explain how you know:

- $9^2 + 7.5^2 = x^2$
- $18 + 15 = x^2$
- $33 = x^2$
- x = about 5.5

5. Provide three possible student responses to the above problem and ask students to identify any correct or incorrect procedures, and explain their decisions.

- a)a correct algebraic response using symbols
- b)an algebraic response using area of the triangle incorrectly
- (A = .5(9)(7.5)) instead of the Pythagorean Theorem
- c)a correct response using a scale diagram

Questioning

When we teach, we ask questions. Are we planning the kinds of questions that we want to ask? The effective use of questions may result in more student learning than any other single technique used by educators.

When designing questions, avoid those that can be answered with “yes” or “no” answers. Remember the levels. Questions written at the knowledge level require students to recall specific information. Recall questions are a necessary beginning, since critical thinking begins with data or facts. There are times when we want students to remember factual information and to repeat what they have learned. At the comprehension level, the emphasis is on understanding rather than mere recall, so we need to ask open-ended questions that are thought provoking and require more mental activity. Students need to show that concepts have been understood, to explain similarities and differences, and to infer cause-and-effect relationships.

Comprehension-level questions require students to think, so we must establish ways to allow this to happen after asking the question.

Ask students to discuss their thinking in pairs or in small groups—the respondent speaks on behalf of the others.

Give students a choice of different answers—let the students vote on the answer they like best—or ask the question to the class, then after the first student responds, without indicating whether that was correct or not, **redirect** the question to solicit more answers from other students.

Ask all students to write down an answer, then select some students to read a few responses.

When redirecting questions (asking several students consecutively without evaluating any) and after hearing several answers, summarize or have the class summarize what has been heard. Or have students individually create (in writing) a final answer to the question now that they have heard several attempts. Lead the class to the correct or best answer to the question (if there is one). Inattentive students who at first have nothing to offer when asked, should be asked again during the redirection of the question to give them a second chance to offer their opinions.

Discourse with the Whole Group

Sometimes it is best to begin discussion by asking more divergent-level questions, to probe all related thoughts and bring students to the awareness of big ideas, then to move towards more convergent questions as the learning goal is being approached. Word the questions well and explain them in other words if students are having trouble understanding. Direct questions to the entire class. Handle incomplete or unclear responses by reinforcing what is correct and then asking follow-up questions. There are times when you want to restate correct responses in your own words, then ask for alternative responses. Sometimes it is important to ask for additional details, seek clarifications, or ask the student to justify a response. Redirect the question to the whole group if the desired response is not obtained. Randomize selection when many hands are waving. Ask a student who is always the first to wave a hand to ask another student for an answer, then to comment on that response. As the discussion moves along, interrelate previous students' comments in an effort to draw a conclusion. It is particularly important to ask questions near the end of the discussion that help make the learning goal clear.

Questioning is a way of getting to assess student progress and an important way to increase student learning. As well, it is a way to get students to think and to formulate and express opinions.

Critical Thinking

Why did ... ?
Give reasons for ...
Describe the steps ...
Show how this ...
Explain why ...
What steps were taken to ... ?
Why do you agree (disagree) with ... ?
Evaluate the result of ...
How do you know that ... ?

Comparison

What is the difference ... ?
Compare the ...
How similar are ... ?
Contrast the ...

Personalized

Which would you rather be like ... ?
What would you conjecture ... ?
Which do you think is correct ... ?
How would your answer compare ...?
What did you try ... ?
How do you feel about ... ?
What would you do if ... ?
If you don't know ... how could you find out?

Cause and Effect Relationship

What are the causes of ... ?
What connection exists between ... ?
What are the results of ... ?
If we change this, then ... ?
If these statements are true, then what do you think is most likely to happen ... ?

Problems

What else could you try ... ?

The diagrams in the problem suggest the following relationship:

x	1	2	3	4	5
y	1	3	6	10	15

What would you say is the y-value when ... ?

Descriptive

Describe ...

Tell ...

State ...

Illustrate ...

Draw (sketch) ...

Define ...

Analyse ...

Levels of Questioning

In recent years, the Nova Scotia Department of Education has administered elementary and junior high mathematics assessments. In an attempt to set standards for these assessments, committees of teachers prepared tables of specifications. These committees also decided that there would be a blend of various complexity levels of questions and determined what percentage of the whole assessment should be given for each level of question. They also agreed that the assessments would use a combination of selected response questions and constructed response questions, some of which might require extended responses, others short responses.

Level 1: Knowledge and Procedure (low complexity)

Key Words

- identify
- compute
- recall
- recognize
- find
- evaluate
- use
- measure

- Level 1 questions rely heavily on recall and recognition.
- Items typically specify what the student is to do.
- The student must carry out some procedure that can be performed mechanically.
- The student does not need an original method of solution.

The following are some, but not all, of the demands that items of "low complexity" might make:

- recall or recognize a fact, term, or property
- recognize an example of a concept
- compute a sum, difference, product, or quotient
- recognize an equivalent representation
- perform a specified procedure
- evaluate an expression in an equation or formula for a given variable
- solve a one-step word problem
- draw or measure simple geometric figures
- retrieve information from a graph, table, or figure

(See page 116 for examples of Level 1 questions.)

Level 2: Comprehension of Concepts and Procedures (moderate complexity)

Key Words

- classify
- organize
- estimate
- explain
- interpret
- compare

- Items involve more flexibility of thinking and choice.
- Questions require a response that goes beyond the habitual.
- The method of solution is not specified.
- Questions ordinarily have more than a single step.
- The student is expected to decide what to do using informal methods of reasoning and problem-solving strategies.
- The student is expected to bring together skills and knowledge from various domains.

The following illustrate some of the demands that items of "moderate complexity" might make:

- make connections between facts, terms, properties, or operations
- represent a situation mathematically in more than one way
- select and use different representations, depending on situation and purpose
- solve a word problem involving multiple steps
- compare figures or statements
- explain and provide justification for steps in a solution process
- interpret a visual representation
- extend a pattern
- retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps

- formulate a routine problem, given data and conditions
- interpret a simple argument

(See pages 116 for examples of Level 2 questions.)

Level 3: Applications and Problem Solving (high complexity)

Key Words

- analyse
- investigate
- formulate
- prove
- explain
- describe
- Questions make heavy demands on students.
- The students engage in reasoning, planning, analysis, judgment, and creative thought.
- The students must think in an abstract and sophisticated way.

The following illustrate some of the demands that items of "high complexity" might make:

- explain relations among facts, terms, properties, or operations
- describe how different representations can be used for different purposes
- perform a procedure having multiple steps and multiple decision points
- analyse similarities and differences between procedures and concepts
- generalize a pattern
- formulate an original problem
- solve a novel problem
- solve a problem in more than one way
- justify a solution to a problem
- describe, compare, and contrast solution methods
- formulate a mathematical model for a complex situation
- analyse the assumptions made in a mathematical model
- analyse or produce a deductive argument
- provide a mathematical justification

(See page 117 for examples of Level 3 questions.)

Recommended Percentages for Testing

- Level 1: 25–30%
- Level 2: 40–50%
- Level 3: 25–30%

Sample Questions

Level 1

1.When $2x + 3y - 12 = 0$ is graphed, the y-intercept will be

- a) 4
- b) -12
- c) - 4
- d) 6

2.If ABC ADE find the length of DE
3.If the vertical height of this pyramid is 15cm, determine the volume.
4.Solve (x+6) = x-9
5.The slope and y-intercept of this line is
6.Find the product of (2x+1)(x-3)
7.Find the factors of x^2 - 8x + 12
8.Determine the missing side:

Level 2
1.In class, your group performed an experiment and collected the following data. The experiment was to determine the distance a ball bearing would roll once it hit the floor after being released from various heights on a elevated ramp.

- a)Plot the graph and describe its shape.
- b)Why is it impossible to obtain a slope for this graph?
- c)If the ball is dropped from a height of 15 cm, how far will it roll?
- d)If the ball rolled 50 cm how high would it have been on the ramp?

2. A reflection in the y-axis can be represented by a mapping rule. Use the mapping rule to determine the equation of the image of the line y = 2x + 1.
3.Use a rectangle and words to demonstrate the connection between line and point symmetry.
4.While exploring transformations on graph paper, Marla transformed the line y=x to the image y=2x. Graph these two lines and explain how the graph shows what transformation has taken place.
5.Illustrate how algebra tiles can be used to determine the factors of 4x2 - 6x.
6.A ladder is being placed against a wall of a house to reach a window sill 4.5 m from the ground. If the foot of the ladder is 1.5 m from the wall, how long must the ladder be?

- Marla solved this question and got the answer 18 m.
- Mark solved this question and got the answer √22.5 m.
- Marty solved this question and got the answer “about 5 m”.

Which answer is the most appropriate for the above problem, and explain why?
7.Determine which of these table of values represents a linear relationships and explain why

- a)x -2 -1 0 1 2 y 10 8 6 4 2
- b)x -2 -1 0 1 2 y 48 24 12 63

Level 3
1.The slope of a wheel-chair ramp must, by regulation, be no more than 5°. You have to construct a ramp o reach a door 3m above the ground, would a 20m ramp be acceptable?

2.If f(x) = 3/5 x – 7, and g(x) = 1/3 x + 5, what is x if f(g(x)) = 5 ?

- Mary's solution:
  - f(g(x)) = 3/5(1/3 x +5) – 7
  - = 1/5 x + 3 – 7
  - = 1/5 x – 4
  - = 1/5(5) – 4
  - = 4
- So, x = - 4, when f(x) = 5.

Has Mary made any calculation errors? Any procedural errors? Justify your answers.

3.Your parents have bought you a cell phone but you are responsible for paying the monthly payments. Here are the three options.

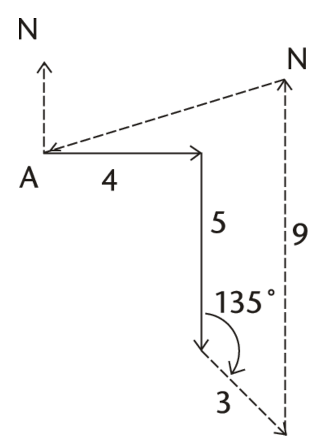
- Plan A: $20 per month includes 200 free minutes of airtime and 8¢ for each additional minute.
- Plan B: $30 per month includes 150 minutes and 5¢ for each additional minute.
- Plan C: $40 per month includes unlimited time.

Use the axis provided to determine under what circumstances each of the other three plans would be the best choice. Justify your answer.

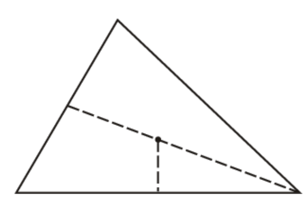
Scoring Open-ended Questions by Using Rubrics

Students should have opportunities to develop responses to open-ended questions that are designed to address one or more specific curriculum outcomes.

Example 1 of "open-ended":
Make up a problem and solve it given the information in this diagram.



Example 2 of "open-ended":
Marla and Ruben bought a triangular piece of property just outside Yarmouth bounded by three roads. They want to position their house so that it is equidistant to each of the roads. Determine and describe the location of the house.



Open-ended questions allow students to demonstrate their understandings of mathematical ideas and show what they know with respect to the curriculum, and they should lead to a solution.

Often, responses to open-ended questions are marked according to a rubric that allows the teacher to assign a level of achievement towards an outcome based on the solution written by the student.

For each individual open-ended question, a rubric should be created to reflect the specific important elements of that problem. Often these individual rubrics are based on more generic rubrics that give examples of the kinds of factors that should be considered. Details will vary for different grade levels, but the basic ideas apply for all levels.

How do you begin thinking about a rubric? Consider this. Let us say that you asked your students to write a paper on a particular mathematician. They handed in their one-page reports, and you began to read them. As you read, you were able to say, Hey, that one is pretty good, this one is fair, and this third one needs a lot of work. So you decide to read them all and put them into the three piles: good, fair, and not so good. A second reading allows you to separate each of the three piles into two more piles: top of the level and bottom of the level. When done, you have six levels, and you could describe the criteria for each of those six levels in order to focus more on the learning outcome. You have created a rubric.

Some rubrics have criteria described for six levels, some five levels, and some four levels. Some people like four levels because it forces the teacher to distinguish between acceptable performance, and below (not acceptable), there is no middle—you either achieve a level 2 (not acceptable work) or level 3 (acceptable).

The first example that follows includes the criteria for a generic four-level rubric, found in the NCTM booklet *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark 1991). In choosing to use this rubric, a teacher would change the generic wording to fit the given problem or open-ended situation. Some teachers like to assign a name to the different achievement levels. For example, they may call the "top level" responses, **exceptional**; the "second level," **good**; the "third level," **not quite**; and the "fourth level," **needs more work**. Many schools simply assign a number rating, usually the top level receiving a 4, then going down, 3, then 2, then 1. Students strive for a level 3 or 4. Some schools assign letters to the categories, giving A to the top level, then B, C, and D.

Achievement Level Criteria
Top Level

- contains a complete response with a clear, coherent, unambiguous, and elegant explanation

- includes a clear and simple diagram
- communicates effectively to an identified audience
- shows understanding of the question's mathematical ideas and processes
- identifies all the important elements of the question
- includes examples and counter-examples
- gives strong supporting arguments
- goes beyond the requirements of the problem

Second Level
- contains a good solid response, with some of the characteristics above, but not all
- explains less elegantly, less completely
- does not go beyond the requirements of the problem

Third Level
- contains a complete response, but the explanation may be muddled
- presents arguments, but they are incomplete
- includes diagrams, but they are inappropriate or unclear
- indicates understanding of the mathematical ideas, but they are not expressed clearly

Fourth Level
- omits significant parts or all of the question and response
- has major errors
- uses inappropriate strategies

A second example of a rubric follows that allows for six achievement levels. This example can be found in the booklet *Assessment Alternatives in Mathematics* (Stenmark 1989), prepared by the Equals staff and the Assessment Committee of the California Mathematics Council Campaign for Mathematics.

Achievement Level Criteria
Exemplary Response
- contains a complete response, with a clear, coherent, unambiguous, and elegant explanation
- includes a clear and simplified diagram
- communicates effectively
- shows understanding of the mathematical ideas and processes
- identifies all the important elements
- may include examples and counter-examples
- presents strong supporting arguments

Competent Response
- contains a fairly complete response, with reasonably clear explanations
- may include an appropriate diagram

- communicates effectively
- shows understanding of the mathematical ideas and processes
- identifies the most important elements of the problem
- presents solid supporting arguments

Satisfactory

- completes the problem satisfactorily, but the explanation might be muddled
- presents incomplete arguments
- includes a diagram, but it is inappropriate or unclear
- understands the underlying mathematical ideas
- uses mathematical ideas effectively

Nearly Satisfactory

- begins the problem appropriately, but fails to complete or may omit parts of the problem
- fails to show full understanding of the mathematical ideas and processes
- may make major computational errors
- may misuse or fail to use mathematical terms
- contains a response that may reflect an inappropriate strategy

Incomplete

- contains an explanation that is not understandable
- includes a diagram that may be unclear
- shows no understanding of the problem situation
- may make major computational errors

Ineffective Beginning

- contains words that do not reflect the problem
- includes drawings that misrepresent the problem situation
- copies part of the problem but without attempting a solution
- fails to indicate which information is appropriate to the problem

No Attempt

- has no evidence of anything meaningful

The top two levels of the above rubric have been titled "Demonstrated Competence" and will get a 6 and a 5. The next two levels called "Satisfactory" will get a 4 and a 3, while the bottom three levels, "Inadequate Response," receive a 2, 1, or 0.

A third kind of rubric that is becoming more popular these days includes more than one domain when assigning the levels. With this type of rubric, the solution attempt will follow the criteria for four levels of achievement, but in four domains: problem solving, understanding concepts, application of procedures, and communication. The teacher assigns levels of achievement for each domain. (See the table on the next page.)

The use of rubrics for assessing open-ended problem situations allows the teacher to indicate how well the student has demonstrated achievement of the outcomes for which the assessment item has been designed in each of these important domains. In reporting to the student or the parent, the teacher can make very clear what it is that the student has not accomplished with respect to full achievement of the outcome(s). Over time, as the outcome(s) is assessed again, progress, or lack thereof, becomes very clear when the criteria are clearly indicated.

Achievement levels can be changed into percentage marks (if that is the desire) by adding together the achievement levels obtained, dividing by the maximum levels obtainable, and changing that ratio to a percentage. For example, Freddie received a level 3, 4, 3, 2, 3, 4, 3, 3, 3, and 2 during the term when open-ended problem-solving opportunities were assigned. He could have obtained a grade of 4 each time, so his ratio is 30 out of a possible 40, giving him a 75 percent mark.

Expectation
Level 1
Level 2
Level 3
Level 4

Problem Solving

- shows no or very little understanding of the mathematical ideas and processes required
- uses no strategies
- shows little understanding
  of the mathematical ideas and processes required
- attempts inappropriate strategy or strategies
- shows an understanding of the mathematical ideas and processes required
- uses some appropriate strategies
- shows a thorough understanding of the mathematical ideas and processes required
- uses appropriate strategies

Understanding Concepts

- demonstrates no understanding of the mathematical concepts required in the problem situation
- demonstrates little understanding of the mathematical concepts required in the problem situation
- demonstrates some understanding of the mathematical concepts required in the problem situation

- demonstrates a thorough understanding of the mathematical concepts required in the problem situation

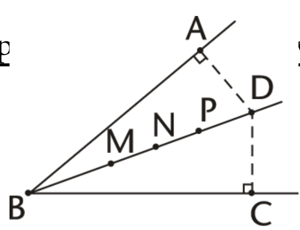
Application of Procedures

- includes a few calculations and/or use of the skills and procedures that may be correct but are inappropriate for the problem situation
- makes several errors in calculations and/or the use of some appropriate skills and procedures
- may make a few errors in calculations and/or the use of skills and procedures
- uses accurate calculations and appropriate skills and procedures

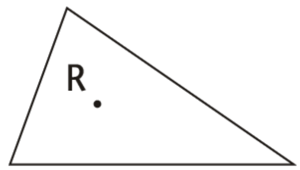
Communication

- includes no appropriate arguments
- includes no appropriate diagrams
- improperly uses words and terms
- makes incomplete arguments
- includes a diagram that is inappropriate or unclear
- may misuse or fail to use mathematical words and terms
- makes a fairly complete response, with reasonably clear explanations and the appropriate use of words and terms
- may include an appropriate diagram
- communicates effectively
- presents some supporting arguments when appropriate
- makes a complete response, with clear, coherent, unambiguous, and elegant explanations and/or use of the words and terms
- illustrates with or presents clear and simplified diagrams
- communicates effectively
- presents strong supporting arguments when appropriate

Examp vel 3 question



	Given $\overline{BD}$ bisects $\angle ABC$ M,N, and P are points anywhere on $\overline{AD}$. Draw perpendiculars from M, N and P, to each of $\overline{AB}$ and $\overline{BC}$. Measure the length of the perpendicular segment. What conjecture might you state about any point on the bisector of an angle in relation to the side of the angle? Describe how you could test your conjecture using D.

	Examine this diagram then determine and describe the location of point R so that it is equivalent distant to the sides of the triangle.

This activity involves the use of a straight edge, and a compass. Students will have to accurately construct the given diagram, draw perpendiculars, from given points, with a compass and measure their lengths. Students will the be asked to conjecture a relationship, then that it and apply it to solve a problem. Ask the students to do the Example 3 above.

	Level 1	Level 2	Level 3	Level 4
Problem Solving	Demonstrates little or no understanding of the requirements to make a conjecture or solve the problem. Makes significant errors or omits steps which leads to no conjecture or wrong conjecture.	Demonstrates some understanding of the requirements to make a conjecture or solve the problem and/or makes significant errors in procedures so that a wrong conjecture is made or that a correct conjecture cannot be made.	Demonstrates a good understanding of the requirements to make a conjecture and to solve the problem. Makes a small error or errors, but still makes a correct conjecture and applies it correctly.	Demonstrates a thorough understanding of the requirements to make a conjecture and to solve the problem draw the diagram accurately measure accurately states a correct conjecture tests the conjecture correctly uses the conjecture to solve the problem correctly for point R.

Understanding Concepts	Demonstrates little or no conceptual understanding. Cannot make a correct conjecture.	Demonstrates some conceptual understanding but does not or cannot make a correct conjecture.	Demonstrates a good understanding of the concepts needed to make the conjecture and/or solve the problem, but has made a small conceptual error like, making the right angle on BD, or not using the three angles to solve the problem.	Demonstrates a thorough understanding of the concepts required. angle bisector perpendicular to a line measures precisely makes a conjecture applies the conjecture
Application of Procedures	Makes major errors that leads to making no conjecture or an incorrect conjecture. Cannot apply the conjecture to solve the problem.	Makes many small errors and some major errors, or makes incorrect conjecture based on errors in procedures.	Makes a few small errors but is able to make a correct conjecture. Possible errors: bisector off perpendicular not quite correct small measurement errors errors in testing the conjecture.	is able to draw an angle and bisect it is able to place points on aline is able to draw a perpendicular to line meausre accurately makes the expected conjecture tests the conjecture applies the conjecture to locate R understands the word “describe”
Communication	Major errors in following directions leads to no conjecture, and/or no solution to the problem.	Major errors in following directions leading to an incorrect conjecture, and/or major error in describing the location of R.	Small errors in following directions but still states a correct conjecture, and/or small error in describing location of the point R.	follows directions completely uses results to formulate a correct and meaningful conjecture. Describes the location of the point R accurately and completely.

Assessment Techniques
Observations

All teachers learn valuable information about their students every day. If teachers have a systematic way of gathering and recording this information, it will allow them to provide valuable feedback about student progress to both students and parents. First, teachers must decide what they are looking for and which students will be observed today. Then the recording method can be decided. *Elementary and Middle School Mathematics Teaching Developmentally,* 5th edition (Van de Walle 2003, pp. 67–68) provides some ideas.

Understanding Students

Super
clear understanding
communicates concepts in
multiple representations
shows evidence of using ideas
without prompting
On Target
understands or is developing well
uses designated models
Not Yet
shows some confusion
misunderstands
models ideas only with help

Use the checklist (above) for several days during the development of a topic or during an activity and record the names of observed students and their achievements.
A generic observation checklist (below), specifically noting what outcomes are being achieved, and to what extent, as the learning develops, may be useful to accompany the above checklist.
Name: Grade:

Not Yet OK Super Comments

Topic: Concept
observe specifics from associated outcomes on the conceptual
development related to the topic
Topic: Procedures
specifics from associated outcomeson the procedural aspects of the
topic to be observed

Specifically, the following checklist shows what this may look like at the grade 10 level, when developing the top of relations.

Name:			**Grade:**	
Relations	**Not Yet**	**OK**	**Super**	**Comments**
represent patterns and relationships in a variety of formats				

predict and justify unknown values using represented patterns and relationships.				
interpret graphs that represent linear data				
interpret graphs that represent non-linear data				

Name:			**Grade:**	
Relations Graphs and equations	**Not Yet**	**OK**	**Super**	**Comments**
construct tables and graphs for given situations				
analyze tables and graphs to determine how change in one quantity affects the related quantity				
determine the equation of lines by obtaining their slopes and y-intercepts from graphs				
sketch graphs of equations using the y-intercept and slope				
solve indirect measurement problems by connecting rate and slope.				

A focussed checklist can be kept for each student during the development of particular topics.

Other examples can be seen in the NCTM booklet, *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark 1991, p. 33).

Daily Observation Sheet

Name:

Date	Activity	Observed Behaviour	Progress Suggestions

Conversations and Interviews

Interviews provide more information about a particular student and what he or she knows, and they provide the opportunity for the teacher to learn more about how the student thinks. Start an interview with questions that the student can be successful with, then ask the student to explain how the answers were obtained, and/or why he or she thinks they are correct. Perhaps ask the student to explain this work to a child in a lower grade level.

- be accepting, but neutral
- avoid cueing or leading the student
- wait silently, do not interrupt
- use prompts like "Show me ...," and "Tell me ..."
- avoid confirming requests for "Am I doing this right?" and "Is this OK?"

Examples of checklists can be seen in the NCTM booklet *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions,* (Stenmark 1991, p. 33).

Name: Week:
Observation and/or Interview Notes
Tell me how you solved the problem.
Is there another way to solve the problem?
Where did you get stuck?

Journal/Log Entries

Writing in mathematics is a very important way to help students clarify their thinking. Have students write explanations, justifications, and descriptions.

Here are some writing prompts for journals or logs:

- I think the answer is ...
- I think this is so, because ...
- Write an explanation for a student in a lower grade or for a student who was absent when this was taught.
- What do I understand? What don't I understand? ...
- I got stuck today because ...
- How do you know you are right?
- Summarize concepts by drawing pictures of things that represent the concept and pictures of things that do not represent the concept.

The best assessment of these writings would be to respond to the students' writing with the intention to develop a written conversation that might lead to clarifying misconceptions and to giving better explanations about concepts or mathematical ideas being developed in the classroom, about how to study or prepare for tests, and so on.

The teacher need only keep a checklist of who has submitted (and when) journal or log entries and to whom he or she has responded.

Self-Assessment

The NCTM booklet, *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark 1991) says that "self-evaluation promotes metacognition skills, ownership of learning, and independence of thought."
When students assess their own learning, the teacher and the student will be able to see

- signs of change and growth in attitudes, mathematical understandings, and achievement
- alignment of how well students are performing with how well they think they are performing
- a match between what the teacher thinks the student can do and what the student thinks the student can do

Some prompts include

- How well do you think you understand this concept?
- What do you believe ...? How you do feel about ...?
- How well do you think you are doing?
- When you work with a group, what are your strengths? What are your weaknesses?

Self-assessment checklists can be developed for each activity in which the teacher wants the students to perform self-assessment. What follows are some suggested yes/no/not sure type questions from which more specific checklists can be developed. These suggestions come from the NCTM booklet *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark 1991). See pages 58 and 59 in that resource for other ideas.

- Sometimes I don't know what to do when I start a problem.
- I like mathematics because I can figure things out.
- The harder the problems, the better I like to work on them.
- I usually give up when a problem is really hard.
- I like the memorizing part of mathematics the best.
- There is more to mathematics than just getting the right answer.
- I think that mathematics is not really useful in everyday living.
- I would rather work alone than with a group.
- I like to do a lot of problems of the same kind rather than have different kinds all mixed up.
- I enjoy mathematics.
- There's always a best way to solve a problem.
- I liked mathematics when I was younger, but now it's too hard.
- Put an "×" on this scale where you think you belong:
  – I am not good at mathematicsI am good at mathematics.