SECTION 2: LANGUAGE AND MATHEMATICS

The ability to read, write, listen, speak, think, and communicate visually about concepts will develop and deepen students' mathematical understanding. It is through communicating about mathematical ideas that students articulate, clarify, organize, and consolidate their thinking. Writing and talking about mathematics makes mathematical thinking visible. Language is as necessary to learning mathematics as it is to learning to read.

As teachers, you must assist students as they learn how to express mathematical ideas, explain their answers, and describe their strategies. You can encourage students to reflect on class conversations and to "talk about talking about mathematics" *(Principles and Standards for School Mathematics,* p. 128). You should ask questions that direct students' thinking and present tasks that help students think about how ideas are related.

Principles and Standards states that instructional programs should enable all students to

- organize and consolidate their mathematical thinking through communication
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- analyse and evaluate the mathematical thinking and strategies of others
- use the language of mathematics to express mathematical ideas precisely

As students progress through the grades they should be developing more fluency in their ability to talk about mathematics and their learning. Their development of language skills is not the primary goal, but students require frequent opportunities to use language to make sense of mathematical ideas through sharing of problem solving strategies and modelling. Students learn through discourse. Communication should include sharing ideas, asking questions, and explaining and justifying ideas. As well, students need experience in writing about their mathematical conjectures, questions, and solutions. "Teachers need to explicitly discuss students' effective and ineffective communication strategies" *(Principles and Standards for School Mathematics,* p. 197).

As teachers, you need to refine your own listening, questioning, and paraphrasing techniques, both to direct the flow of learning and to provide models for student dialogue. By asking questions that require students to articulate and extend their thinking, you are able to assess students' understanding and make appropriate instructional decisions.

As students progress into middle grades they should be expected not only to present and explain a strategy used to solve a problem but also to compare, analyse, and contrast the usefulness, efficiency, and elegance of a variety of strategies. Students must be encouraged to provide explanations of mathematical arguments and rationales, as well as procedural descriptions or summaries. The use of oral and written communication provides students opportunities to

- think about solving problems
- develop explanations
- acquire and use new vocabulary and notation
- explore various ways of presenting arguments
- critique and justify conjectures
- reflect on their own understanding as well as that of others

"Teachers should build a sense of community ... so students feel free to express their ideas honestly and openly, without fear of ridicule" *(Principles and Standards for School Mathematics,* p.269).

Classroom Strategies

According to provincial curricula, students must relate to and respond to a wide variety of experiences using many communicative forms of representation to express their work. Teachers need to utilize strategies that will allow students to process what they are learning, organize their ideas in ways that are meaningful to them, and to reflect on and extend their learning.

The integration of the five mathematics strands from the *Atlantic Canada Mathematics Curriculum* (Number Concepts/Number and Relationship Operations, Patterns and Relations, Shape and Space, Data Management, and Probability) with the three language strands from the *Atlantic Canada English Language Arts Curriculum* (Speaking and Listening, Reading and Viewing, and Writing and Other Ways of Representing) provides the richness and diversity in curriculum and instruction that will enable students to confidently explore mathematical concepts and ideas . The essence of the three language strands can be found in the *Atlantic Canada English Language Arts Curriculum: Grade 10* on pages 119 – 152. In this document, Appendix O: *Outcomes Framework to Inform Professional Development for Teachers of Reading in the Content Areas* identifies the expectations of all content teachers with respect to literacy.

To help teachers deliver the prescribed curriculum, this section identifies classroom strategies that support teachers as they develop the vocabulary of mathematics, initiate effective ways to maneuver informational text, and

encourage students to reflect on what they have learned. Along with a suggestion of how to use the strategy is a mathematical application. When attempting to use these strategies, teachers are reminded of the suggestions found in *Secondary Science: A Teaching Resource*

- start small—try something that feels comfortable and build competence with time
- network with others
- take advantage of professional development opportunities

At this time it is important to stress that when teachers use these strategies in the instructional process or as assessment tasks, the expectations for students must be made explicitly clear. The student's understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concern.

Name of Strategy	Before	During	After	Assessment
1. Concept Circle	X	X	X	X
2. Frayer Model	X	X	X	X
3. Concept Definition Map	X	X	X	X
4. Word Wall	X	X	X	
5. Three Read		X	X	
6. Graphic Organizer	X	X	X	X
8. K-W-L	X		X	X
7. Think-Pair-Share	X	X		
9. Think Aloud	X	X		
10. Academic Journal Mathematics		X	X	
11. Exit Cards			X	X

1. Concept Circle

A concept circle is a way for students to conceptually relate words, terms, expressions, etc. As a before activity, it allows students to predict or discover relationships. As a during or after activity, students can determine the missing concept or attribute, or identify an attribute that doesn't belong.

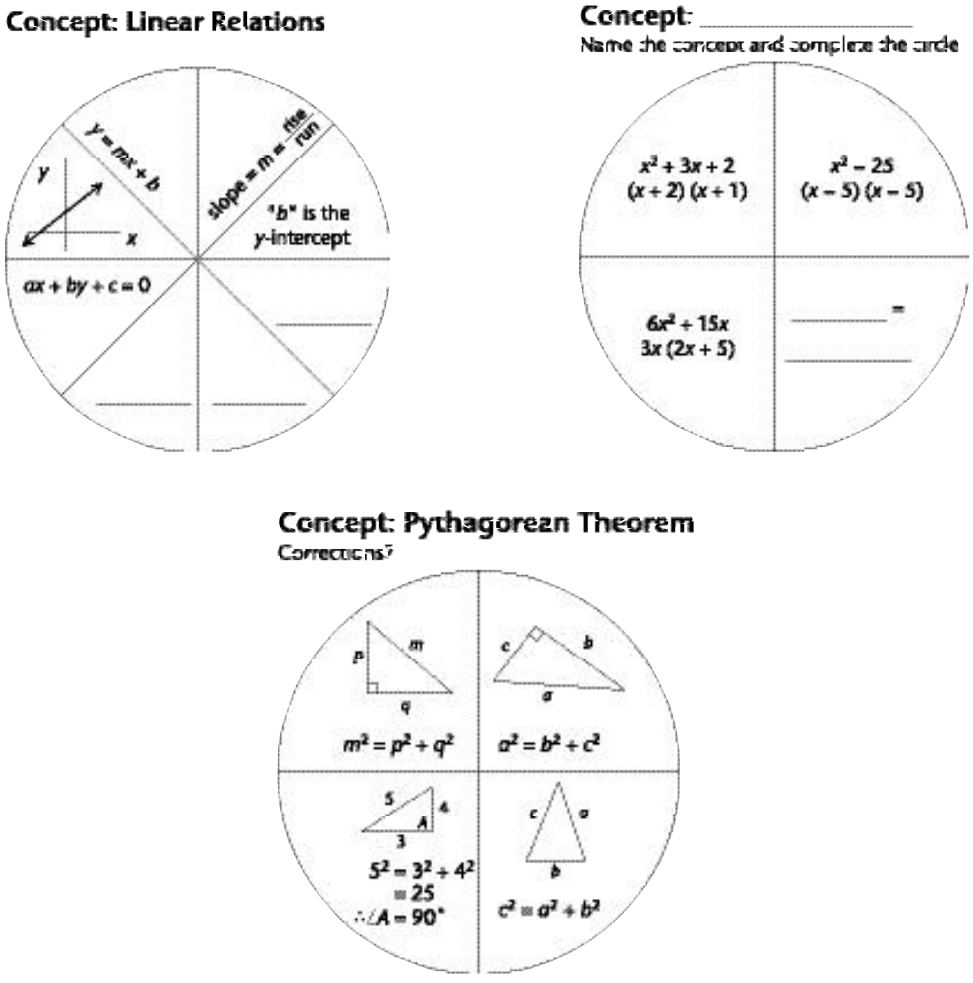
The following steps illustrate how the organizer can be used.

- draw a circle with the number of sections needed
- choose the common attributes and place them in the sections of the circle
- have students identify the concept common to the attributes

This activity can be approached in other ways.

- supply the concept and some of the attributes and have students supply the missing attributes
- insert an attribute that is not an example of the concept and have students find the one that does not belong and justify their reasoning

Concept Circle

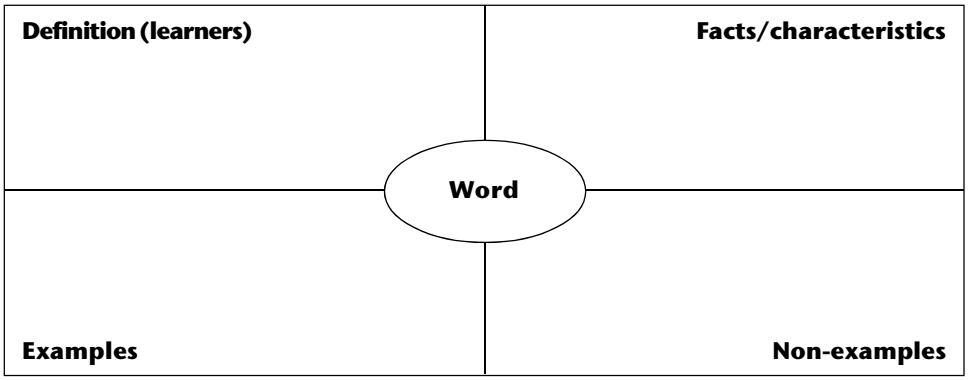


2. The Frayer Model

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing the main characteristics
- providing examples and non-examples of the word or concept

It is especially helpful to use with a concept that might be confusing because of its close connection to another concept. The following is a rendition of the template found in Appendix G*.



The following steps illustrate how the organizer can be used.

- Display the template for the Frayer Model and discuss the various headings and what is being sought.
- Model how to use this map by using a common word or concept. Give students explicit instructions on the quality of work that is expected
- Establish the groupings (e.g., pairs) to be used and assign the concept(s) or word(s)
- Have students share their work with the entire class

This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept or a different word or concept could be assigned.

The Frayer Model

Essential Characteristics

- a rectangular arrangement of numbers in rows and columns

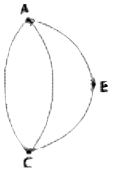
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots \\ b_{11} & b_{12} & b_{13} & \dots \\ c_{11} & c_{12} & c_{13} & \dots \end{bmatrix}$$

Nonessential Characteristics

- number of rows and columns might be the same (square matrix) or might be different
- only some matrices can be multipled
- a network can be modeled with a matrix
- as digraph can be drawn to represent a matrix, and vice versa

Matrix

Examples



$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$\underset{2 \times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}} \times \underset{3 \times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}} \underset{2 \times 2}{\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}}$$

Nonexamples

a) $\begin{matrix} 2 & 3 \\ 1 & 5 \end{matrix}$

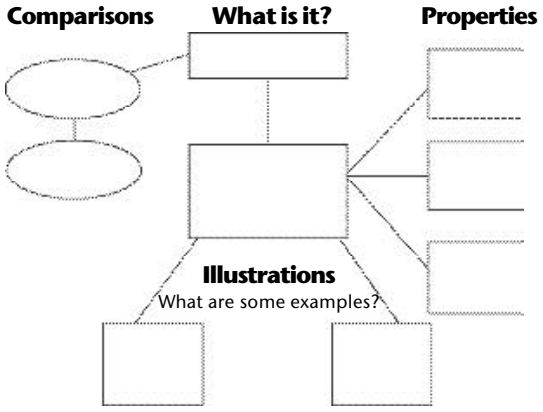
b) $\frac{1}{2}$

c) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 5 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = ?$

3. Concept Definition Mapping

The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to

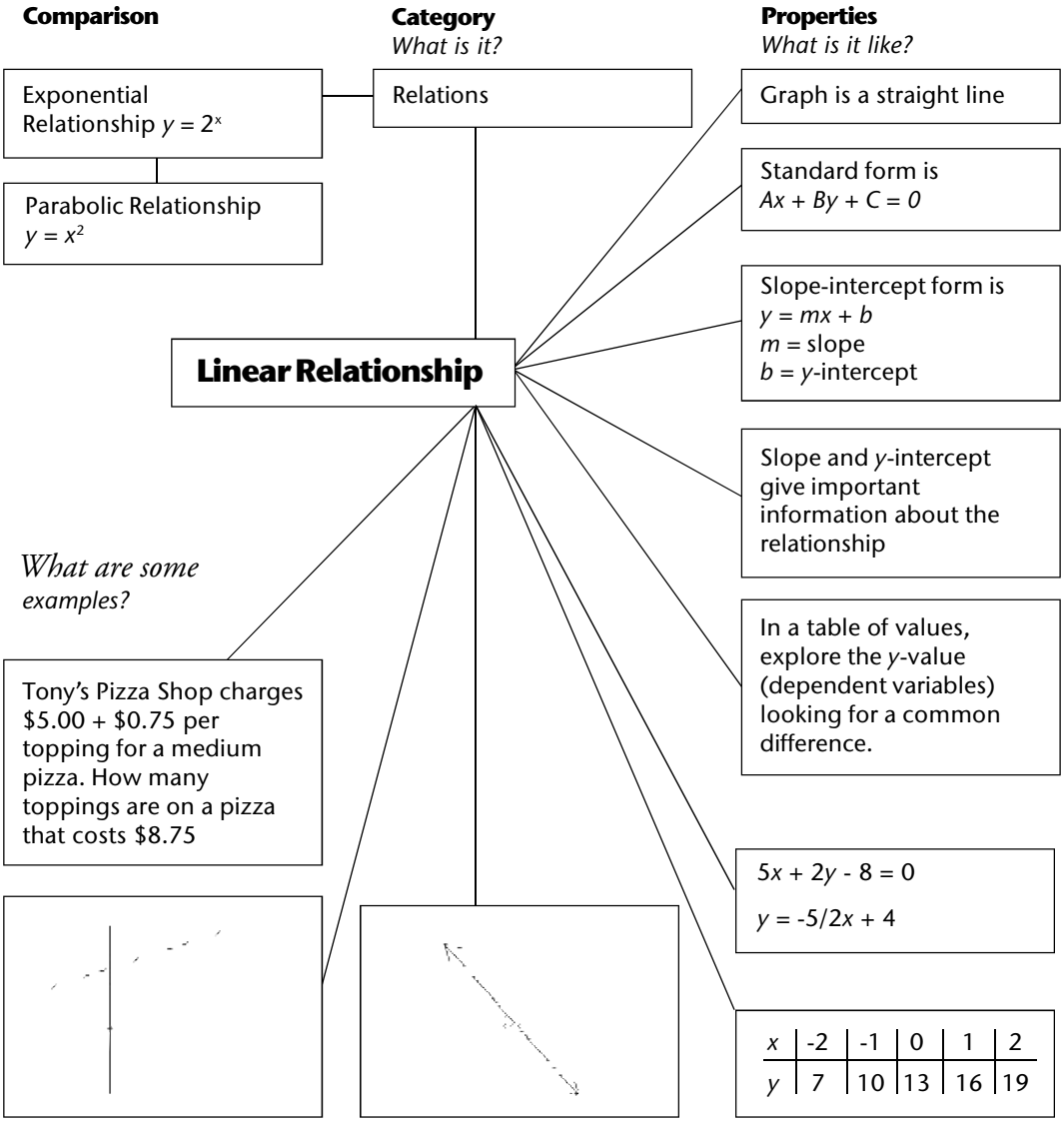


identify properties, characteristics, and examples of the concept. The following is a rendition of the template found in Appendix H.

The following steps illustrate how the organizer can be used.

- Display the template for the concept definition map.
- Discuss the different headings, what is being sought, and the quality of work that is expected.
- Model how to use this map by using a common concept.
- Establish the concept(s) to be developed.
- Establish the grouping (e.g., pairs) and materials to be used to complete the task.
- Complete the activity by having the student write a complete definition of the concept.

Encourage students to refine their map, as more information becomes available.



4. Word Walls

A mathematics word wall is based upon the same principle as a reading word wall found in many classrooms. It is an organized collection of words that is prominently displayed in the classroom and helps teachers teach the language of mathematics. A word wall could be dedicated to a concept, big idea or unit in the mathematics curriculum. Words are printed in bold, block letters on a card and then posted on the wall or bulletin board.

Depending upon the grade level, illustrations placed next to the word on the wall would add to the students' understanding. Students may also elaborate on the word in their journals by illustrating, showing an example, and using the word in a meaningful sentence or short paragraph. Students can each be assigned a word and its illustration to display on the word wall. Room should be left to add more words and diagrams as the unit or term progresses.

As a new mathematical term is introduced to the class, students can define and categorize the word in their Mathematics Journals under an appropriate unit of study. Then the word can be added to the Mathematics Word Wall so students may refer back to it as needed. Students will be surprised about how many words fall under each category and how many new words they learn to use in mathematics.

Note: The word wall is developed one word at a time as the new terminology is encountered.

To set up a word wall:

1. Determine the key words that students need to know or will encounter in the topic or unit.
2. Print the word in large block letters and add the appropriate illustrations.
3. Display the card when appropriate
4. Regularly review the words as a warm-up or refresher activity.

5. Three Read Strategy

Using this strategy, found in the Nova Scotia Department of Education's document Towards a Coherent Mathematics Program (2002), the teacher encourages students to read the problem three times before they attempt to solve it. There are specific purposes of each reading.

First Read

The students try to visualize the problem in order to get an impression of the overall context of the problem. They do not need specific details at this stage, only a general idea so they can describe the problem in broad terms.

Second Read

The students begin to gather facts about the problem to make a more complete mental image of the problem. As they listen for more detail, they focus on the information to determine and clarify the question.

Third Read

The students check each fact and detail in the problem to verify the accuracy of their mental image and to complete their understanding of the question.

During the Three Read strategy, the students discuss the problem including any information needed to solve the problem. Reading becomes an active process that involves oral communication among students and teachers and also written communication as teachers encourage students to record information and details from their reading as well as to represent what they read in other ways with pictures, symbols, or charts. The teacher facilitates the process by posing questions that ask students to justify their reasoning, to support their thinking, and to clarify their conclusions.

6. Graphic Organizer

A graphic organizer can be of many forms: webs, charts, diagrams etc. Graphic organizers use visual representations as an effective tool to do such things as

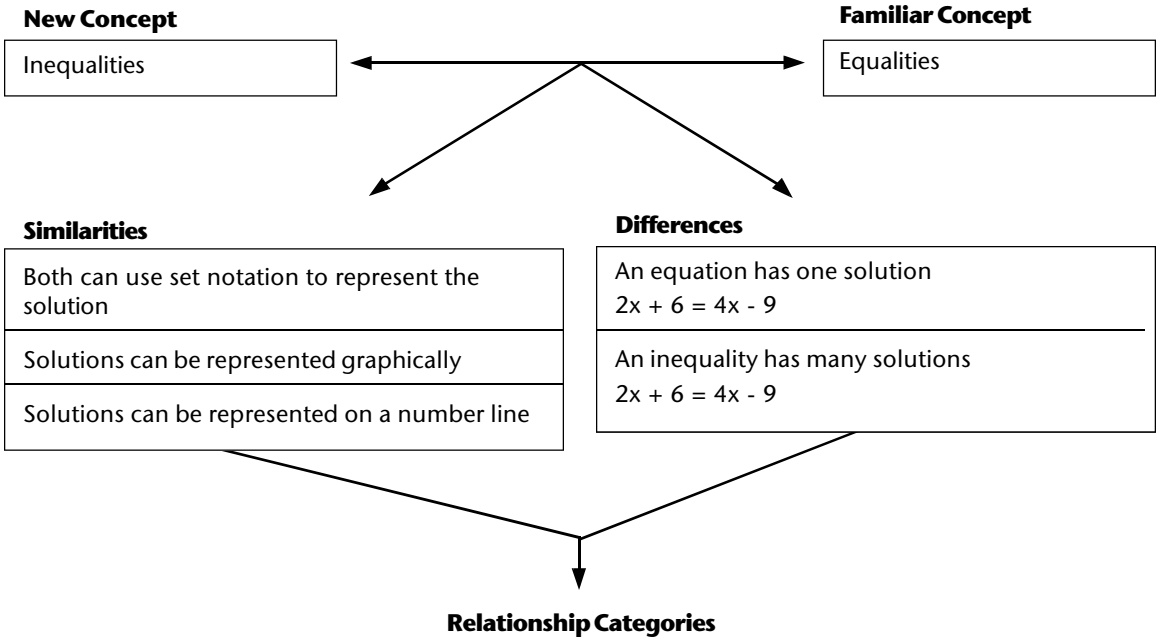
- activate prior knowledge
- make connections
- organize
- analyse
- compare and contrast
- summarize

The following steps illustrate how the organizer can to be used.

- Present a template of the organizer and explain its features.
- Model how to use the organizer, being explicit about the quality of work that is expected.
- Present various opportunities in the classroom and as assessment tools for students to use graphic organizers

Students should be encouraged to used graphic organizers on their own as a way of organizing their ideas and work.

Analogy Graphic Organizer



7. K-W-L (Know/Want to Know/Learned)(Ogle,1986)

K-W-L is an instructional strategy that guides students through a text and uses a three column graphic organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students record what they have learned in the L column. The K-W-L strategy has several purposes.

- obtain a student's prior knowledge of a topic
- give a purpose to the reading
- help a student monitor his/her comprehension

Know	Want to Know	Learned

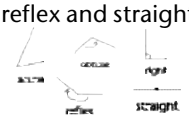
The following steps illustrate how K-W-L can be used.

- Present a template of the organizer to the students, explain its features, and be explicit about the quality of work that is expected.
- Ask them to fill out the first two sections: what they know and what they want to know before proceeding.

- Check the first section for any misconceptions in thinking or weaknesses in vocabulary.
- Have the students read the text, taking notes as they look for answers to the questions they posed.
- Have students complete the last column to include the answers to their questions and other pertinent information.
- Discuss this new information with the class and address any questions that were not answered.

Variations on this strategy are explained in *Secondary Science: A Teaching Resource*, pages 2.9–2.15.

K-W-L Concept Sum of the Interior Angles of a Quadrilateral

Know	Want to Know	Learned
• bearing can be represented by an angle • angles are the union of two rays at a common endpoint called the vertex of the angle  • angles can be acute, obtuse, or right, also reflex and straight  • angles are measured in degrees ... there are 360° in a circle • to measure an angle, use a protractor and measure the number of degrees from one side of the angle (start at zero) to the other.	• what is a bearing? • how do we measure bearings? • how are they used? • what is a vector? • how do vectors relate to bearings?	• bearing is the angle of directin meausred from the north • bearings are oftenused along with distance • vectors are often used for direction and distance - the direction is indicated by the arrowhead on the vector, or by a given bearing, and the distance is a related number • vectors can be representatinos of bearings, examples:  vector: 3km east  vector: 2km northeast  bearing: 225° bearing: 2km, 225°

8. Think-Pair-Share

Think-Pair-Share is a learning strategy designed to encourage students to participate in class and keep them on task. It focusses students' thinking on specific topics and provides the students with an opportunity to collaborate and engage in meaningful discussion (Van de Walle, 2000).

- First, teachers ask students to think individually about a newly introduced topic, concept, or problem. This provides essential time for each student to

collect their thoughts and focus their thinking. (read Focus C, pg. 216 Nelson, and complete KWL for learning vectors.)

- Second, each student pairs with another student and together the partners discuss each other's ideas and points of view and review your KWL with another student. Students are more willing to participate because they don't feel the peer pressure that is involved when responding in front of the class. Teachers ensure that sufficient time allows each individual student to voice his or her views and opinions. Students use this time to talk about personal strategies, compare solutions, or to test ideas or understandings (in this case) with their partners. This forces students to make sense of the problem or information in terms of prior knowledge.
- Third, each pair of students shares with the other pairs of students in a large group discussion. In this way, each student has the opportunity to listen to all the ideas and concerns discussed by the other pairs of students. Teachers point out similarities, overlapping ideas, or discrepancies among the pairs of students and facilitate an open discussion to expand upon any key points or arguments they wish to pursue.

9. Think Aloud

Think-aloud is a self-analysis strategy that allows students to gain an insight into the thinking process of a skilled reader as he/she works through a piece of text. Thoughts are verbalized and meaning is constructed around vocabulary and comprehension. It is a useful tool for such things as brainstorming, exploring text features, and constructing meaning when solving problems. When used in mathematics, it can reveal to teachers the strategies that are part of a student's experience and those that are not.

The think aloud process will encourage students to use the following strategies as they approach a piece of text.

- connect new information to prior knowledge
- develop a mental image
- make predictions and analogies
- self-question
- revise and fix-up as comprehension increases

The following steps illustrate how to use a think aloud.

- Explain that reading in mathematics is important and requires students to be thinking and trying to make sense of what they are reading.
- Identify a comprehension problem or piece of text that may be challenging to students, have students read it quietly while the teachers reads it aloud.

- While reading, teachers can model the process by verbalizing what they are thinking, what questions they have, and how they would approach a problem.
- This process should then be modelled a second time but have a student read the problem and do the verbalizing.
- Once students are comfortable with this process, then a student should take a leadership role.

10. Academic Journals—Mathematics

An academic journal in mathematics is an excellent way for students to keep personal work and other materials that they have identified as being important for their personal achievement in mathematics. The types of materials that students would put in their journal would be

- strategic lessons—lessons that they would identify in being pivotal as they attempt to understand mathematics
- examples of problem-solving strategies
- important vocabulary

Teachers are encouraged to allow students to use these journals in an assessment. This way it will emphasize to the student that the material that is to be placed into their journal has a purpose. Also, teachers would probably only mark these journals on the basis of how students are using them and whether or not they have the appropriate entries.

Math Journals

The goal of writing in mathematics is to provide students with opportunities to explain their thinking about mathematical ideas and then to re-examine their thoughts by reviewing their writing. Writing will enhance students' understanding of math as they learn to articulate their thought processes in solving math problems and learning mathematical concepts.

11. Exit Cards

Exit cards are a quick tool for teachers to become better aware of a student's understanding. They are written student responses to questions that have been posed in class or solutions to a problem-solving situation. They can be used at the end of a day, week, lesson, or unit. An index card is given to each student with a question that promotes understanding on it and the student must complete the assignment before they are allowed to "exit" the classroom. The time limit should not exceed 5–10 minutes and the student drops the card into some sort of container on the way out. The teacher now has a quick assessment of a concept that will help teachers as they plan instruction.

Grade 10 Mathematics Language

Number Sense

- Absolute Value
- Continuous data
- cube Root
- Discrete data
- Inequality
- Integers
- Irrational numbers
- Matrices
- Matrix
- Perfect Square
- Perfect Cube
- Principal square root
- Pythagorean Triple
- Radical
- Radicand
- Rational numbers
- Real numbers
- Set notation
- Set of real numbers
- Solution set
- Square root
- Subset of real numbers
- Undefined

Operations

- Algebraic expression
- Additive Inverse
- Associative
- Binomial
- Commutative
- Distributive
- Exponent laws
- Identity
- Identity property
- Integral exponents
- Inverse
- Inverse property
- Matrix operations
- Monomial
- Multiplicative Inverse
- Polynomial
- Product matrix
- Rational numbers
- Scalar
- Scalar multiplication
- Square roots
- Trinomial

Patterns & Relationships

- Adjacency Matrix
- common Factoring
- Degree of equations
- Difference of Squares
- Digraphs
- Dimensions (of a matrix)
- Domain
- Edge
- Element (of a matrix)
- Equation
- Even vertex
- Exponential relation
- Factoring
- Factors
- Function
- Identity
- Intersection Point
- Linear
- Major Diagonal (of a matrix)
- Mapping Notation
- Modeling
- Network
- Network Graph
- Odd vertex
- Parabolic curve
- Perfect Square
- Quadratic
- Range
- Rise
- Run
- Sequence
- Slope of a line
- Slope
- Slope y-intercept form of equation
- Square matrix
- Tree diagram
- Verify
- Vertical Line Test
- X-intercept
- Y-intercept
- Single-variable equation
- Independent variable
- Dependent variable

Measurement

- Accuracy
- Angle of Elevation
- Bearing angle
- Capacity
- Cone
- Congruent triangles
- Cosine
- Cylinders
- Economy rate
- Lateral Face
- Lateral Surface
- Metric conversion
- Precision
- Prisms
- Pyramid
- Rates
- Ratios
- Resultant Vector
- scale Factor
- Similar triangles
- Sine
- Slant Height
- Slopes
- Sphere
- Tangent
- Trigonometric Ratios
- Vector
- Volume

Geometry

- Adjacent Side
- Altitude
- Angle Bisector
- Apothem
- ASA/SAS/SSS
- Centre of Gravity
- circumcentre
- Circumscribed Circle
- Coincident
- Collinear
- Congruence
- Congruent triangles
- Deductive reasoning
- Dilatations
- Exterior Angle
- Heptagon
- Hexagon
- Hypotenuse
- Incentre
- Inscribed Circle
- Inductive reasoning
- Interior Angle
- Line Symmetry
- Median (of a triangle)
- Minimum sufficient conditions
- Octagon
- Opposite Side
- Order of Rotational Symmetry
- Orientation of image
- Pentagon
- Perpendicular Bisector
- Polygon
- Pythagorean Theorem
- Reflections
- Regular Polygon
- Rigidity
- Rotations
- Rotational Symmetry
- Similarity
- Transformations
- Translations

Data Management

- Bin
- Box-and-whisker plot
- Broken-Line Graph
- Controlled variables
- Correlation – strong and weak
- Correlation Coefficient
- Curve of best fit
- Data analysis
- Dependent variable
- Dispersion
- Distribution
- Extrapolate
- Extremes – upper and lower
- Frequency Polygon
- Frequency Table
- Histogram
- Independent variable
- Interpolate
- Line of best fit
- Mean
- Median
- Median-median Line
- Mode
- Negative correlation
- Normal distribution
- Outlier
- Positive correlation
- Prediction
- Precision
- Quartile – upper and lower
- Range
- Regression
- Scatter plot
- Significant digits
- Standard Deviation
- Stem-and-Leaf Plot
- Summary Point

Probability