Chapter 6 – Trees

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Some Definitions

- Nodes with no children are leaves: (C,E,F,H,I), they are also called external nodes. Nodes which are not leaves are called internal nodes
- Nodes with the same parents are siblings: (B,C,D,E) and (G,H)
- A path from node n_i to node n_j is the sequence of directed edges from n_i to n_j
- The level or depth of a node n_i is the number of edges from the root to n_i. The depth of the root is 0

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Some Definitions (cont'd)

- The height of a node n_i is the length of the longest path from n_i to a leaf. The height of a leaf node is 0
- The height of a tree is equal to the height of the root

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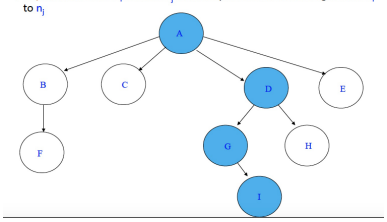
Path

Some Definitions

❑A path from node n_i to node n_j is the sequence of directed edges from n_i to n_j



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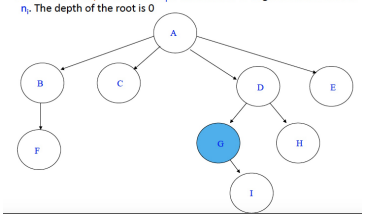
Level (depth) of a node

Some Definitions

❑The level or depth of a node n_i is the number of edges from the root to n_i. The depth of the root is 0



Depth of D: 1

Depth of G: 2

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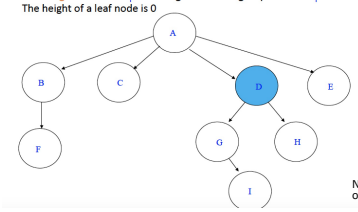
Height of a node

Some Definitions

❑The height of a node n_i is the length of the longest path from n_i to a leaf. The height of a leaf node is 0



Height of D: 2

Height of G: 1

Height of A: 3

Note: AE: From A to E the length of the path is 1.

But AE is not the Longest path from A to a leaf!

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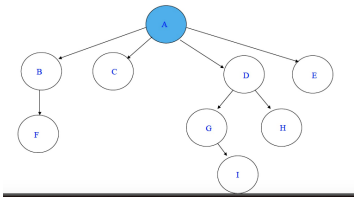
Height

Some Definitions

❑The height of a tree is equal to the height of the root



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Binary Tree

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Binary Trees – An Informal Definition

- A binary tree is a tree in which no node can have more than two children
- Each node has 0, 1, or 2 children

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Binary Trees – A Recursive Definition

1. An empty structure is a binary tree
2. If T_1 and T_2 are binary trees, then the structure whose root has as its children the roots of T_1 and T_2 is also a binary tree
3. Only structures generated by rules 1 and 2 are binary trees

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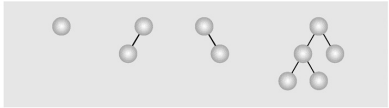
Trees vs. Binary Trees

- No node in a binary tree may have more than 2 children, whereas there is no limit on the number of children of a node in a tree

FIGURE 6.4 Examples of binary trees.



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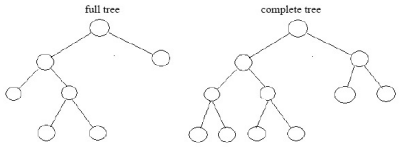
Types of Binary Trees

- A binary tree in which each node has exactly 0 or 2 children is called a full binary tree – there are no degree 1 nodes
- A complete binary tree is a tree which is completely filled, with the possible exception of the bottom level, which is filled from left to right



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Properties of BT

- Min/Max number of nodes in a binary tree whose height is *h*
- Min/Max height in a binary tree with n nodes
- Min/Max number of leaves/internal nodes/in a binary tree whose height is *h*

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Next: find these max/min one by one

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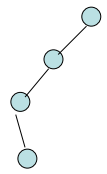
Minimum number of nodes with Height =h

– Minimum number of nodes in a binary tree whose height is *h = 3*



At least one node at each level

➔ Minimum number of nodes is *4 = 3 +1*

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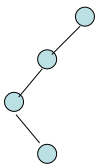
Minimum number of nodes with Height = h

- Minimum number of nodes ?
  – Minimum number of nodes in a binary tree whose height is *h*



At least one node at each level

➔ Minimum number of nodes is *h* + 1

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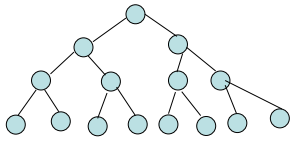
Max number of nodes with Height =h

- Maximum number of nodes ?
  – Maximum number of nodes in a binary tree whose height is *h*
  – *h =3*
  – 2^0
  – 2^1
  – 2^2
  – 2^3



A complete binary tree

$2^0 + 2^1 + 2^2 + 2^3 = 15 = 2^4 - 1$

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Max # of Nodes for a binary tree with Height h

- h= 0 1 node
- h=1 3 nodes = 1 (at level 0)+ 2 (at level 1)
- h=2 7 nodes = 1 (at level 0)+ 2 (at level 1) + 4 (at level 2)
- h=3 15 nodes = 1 (at level 0)+ 2 (at level 1) + 4 (at level 2) + 8 (at level 3)

- For any h, $2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^h = 2^{h+1} - 1$

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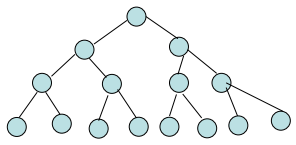
Max number of nodes with Height =h

- Maximum number of nodes ?
  - Maximum number of nodes in a binary tree whose height is *h*
  - *h*
  - 2^0
  - 2^1
  - 2^2
  - 2^h



A complete binary tree

$2^0 + 2^1 + 2^2 + \ldots + 2^h = 2^{h+1} - 1$

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Max/Min height in a binary tree with n nodes

- Maximum height ?
  - Maximum height in a binary tree with n nodes
- Minimum height ?
  - Minimum height in a binary tree with n nodes

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Maximum height in a binary tree with n nodes

- Maximum height ?
  - Maximum height in a binary tree with n nodes

n=1 → h =0
n=2 → h =1
n=3 → h =2 what does the tree's shape look like?

Max h = n-1

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Min height in a binary tree with n nodes

- Minimum height ?
  - For a tree of height h, the number of nodes n satisfies the following:

$n <= 2^{h+1} - 1$
$n+1 <= 2^{h+1}$
$\log_2(n+1) <= \log_2 2^{h+1}$
Thus $h >= \log_2(n+1) - 1$

The minimal height of a binary tree with n nodes is: $\log_2(n+1) - 1$ (round up)

- example, n = 15, min height = 3
- n = 14, min height = 3
- n =7, min height = 2

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Max/Min number of external nodes (or leaves)?

- Maximum number of leaves in a binary tree with height h
- Minimum number of internal nodes?
  - Maximum number of internal nodes in a binary tree with height h

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Max number of external nodes (or leaves)?

2^h

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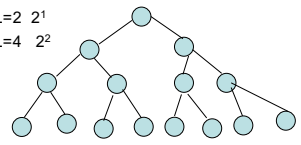
Max number of leaves when Height =h

– Maximum number of leaves in a binary tree with height h

h =0 L=1 2^0
h =1 L=2 2^1
h =2 L=4 2^2



– 2^h

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Max number of non-leaves for in a binary tree with height h?

2^h-1

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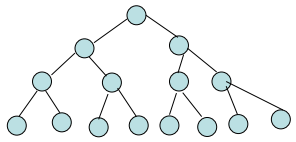
Max number of non-leaves for in a binary tree with height h?

- Maximum number of nodes ?
  - Maximum number of nodes in a binary tree whose height is *h*
  - *h*
  - 2^0
  - 2^1
  - 2^{h-1}



A complete binary tree

$2^0 + 2^1 + .. + 2^{h-1} = 2^h - 1$

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Maximum number of internal nodes in a binary tree with height h

- Minimum number of internal nodes?
  - Maximum number of internal nodes in a binary tree with height h

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Minimum number of internal nodes in a binary tree with height h

- Minimum number of internal nodes?
  - Maximum number of internal nodes in a binary tree with height h

  - h

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