Journal of Statistical Physics, Vol. 50, Nos. 1/2, 1988

Initiation of Damage in the Kauffman Model

Martin Corsten[1] **and Peter Poole**[1]

Received September 9, 1987

An alternate method is presented for initiating damage to the Kauffman model. A numerical study suggests that this modification increases the efficiency of generating damage clusters without changing their fractal properties.

KEY WORDS: Cellular automata; Kauffman model; damage.

There has been recent interest in random cellular automata, in particular in the Kauffman model.(1–3) In the Kauffman model each site on a lattice has a spin, which may be "up" or "down," and there is also a set of rules unique to that site. These rules, which are generated randomly, determine the state of the spin at any given site based on the spin states of neighboring sites. Therefore, on a square lattice each site has 2^4, or 16, rules.

The concept of "damage" in the Kauffman model is a major area of study.(4,5) If two lattices differ only by the orientation of a single spin at t_0, the lattices will in most cases become less and less similar as $t \to \infty$. In studies of damage a third lattice is generated with spin "up" wherever the two lattices have different spin states and spin "down" wherever the two lattices have the same spin state. The set of "up" spins is known as the "damage cloud" and it is the properties of this cloud that are of interest in this paper. In a recent report, Stauffer(4) described the damage cloud as a percolation-type cluster and introduced three critical exponents to describe its properties: d_t, d_{act}, and d_{tot}. These fractal dimensions are defined by

$$\langle t \rangle \propto L^{d_t} \tag{1}$$

where $\langle t \rangle$ is the average time for damage to hit a boundary at a distance L from the initial damage site:

$$\langle M_{\text{act}} \rangle \propto L^{d_{\text{act}}} \tag{2}$$

[1] Saint Francis Xavier University, Antigonish, Nova Scotia, Canada B2G 1CO.

0022-4715/88/0100-0461$06.00/0

where $\langle M_{\text{act}} \rangle$ is the average actual damage (i.e., damage at the time step when the damage touches the edge of the system); and

$$\langle M_{\text{tot}} \rangle \propto L^{d_{\text{tot}}} \tag{3}$$

where $\langle M_{\text{tot}} \rangle$ is the average total damage (i.e., total set of sites damaged up to a given time step).

It has been suggested that the notion of damage in the Kauffman model can be used to describe how complex systems are affected by small errors within them. Some examples of relevant problems include the mutation of a gene in DNA, the damage to a chip within a computer, or the infection of a cell within an organism.

A modification that can be made to the generation of damage in the Kauffman model is to replace the initial damage created by changing the *spin* at one site to an initial damage created by changing the 16 *rules* at one site. This modification may be motivated as follows: when genes are damaged, for example, they are usually damaged irrevocably. Likewise, computer chips and body cells are usually damaged in a permanent way. Changing a spin only damages a site temporarily, in that the site can revert to its original form at a subsequent time step and perhaps never indicate by its subsequent behavior that it was previously damaged. However, changing a rule alters a site in a permanent way, and is thus more realistic.

The damage structures produced by the change of a rule ("rule-change" clouds) have cluster properties similar to those of the "spin-change" clouds. A plot of p, which is the fraction of rules which give an "up" value to the spin, versus the order parameter $\langle M_{\text{tot}} \rangle$ shows the following characteristic behavior: above a value $p = p_c$ the damage spreads to infinity; below p_c, then, damage does not. The plot suggests a value of $p_c = 0.29$, which is virtually identical to that quoted by Stauffer[4] for spin-change clouds on the square lattice.

When the fractal dimensions d_t and d_{act} were measured for the rule-change damage clouds, it was found that the exponents were the same for both rule-change and spin-change clouds. That is, initiating damage by changing a rule and changing a spin creates damage clouds with identical fractal properties. Our fractal dimensions are calculated to be $d_t = 1.7$ and $d_{\text{act}} = 1.6$, in good agreement with the values quoted by Stauffer,[4,5] with typical error bars of ± 0.1.

Since rule-change initiation of damage in the Kauffman model gives clouds with the same fractal properties as the spin-change clouds, it is tempting to dismiss this modification with the argument that is does not provide any new information. However, there is an advantage to using the rule-change-initiated damage: it eliminates some of the attrition that occurs in the development of damage clouds. At p_c the clouds under investigation are those that span the system, that is, the "infinite" clouds. In many cases, for both spin-change and rule-change clouds, the damage does not span the system, and those clouds that do not span are ignored in determining the critical properties. However, in the rule-change case the initial damage is more permanent; it gives the cloud a greater probability of spanning the system. In *all* of the trials we performed the rule-change cloud spanned the system significantly more often than would a comparable spin-change cloud.[4] For example, at $L = 150$ a spanning cloud was achieved in 16 out of 25 rule-change attempts, almost twice as many as would be generated from a spin-change initial damage.

In summary, the effect of using the rule-change modification in the generation of damage within the Kauffman model is that it reduces the attrition in the growth of the damage clouds *without changing their fractal properties*. This results in an efficient use of computing time.

ACKNOWLEDGMENTS

We would like to thank Naeem Jan and Dietrich Stauffer for their helpful suggestions. This work was supported by the Natural Sciences and Engineering Research Council of Canada and the Research Council of Saint Francis Xavier University.

REFERENCES

1. S. A. Kauffman, *J. Theor. Biol.* **22**:437 (1969).
2. S. A. Kauffman, *Physica D* **10**:145 (1984).
3. S. A. Kauffman, in *Disordered Systems and Biological Organization*, E. Bienstock, F. Fogelman Soulie, and G. Weisbuch, eds. (Springer-Verlag, Heidelberg, 1986).
4. D. Stauffer, Proceedings of the Conference "The Physics of Disordered Systems," Bar Ilan University, *Phil. Mag.* (1987), to be published.
5. L. De Arcangelis, *J. Phys. A* **20**:L369 (1987).

Communicated by D. Stauffer