Comment on "Spreading of Damage: An Unexpected Disagreement between the Sequential and Parallel Updatings in Monte Carlo Simulations"

In the Letter by Nobre, Mariz, and Sousa [1], the authors describe their investigation of damage spreading in the Ising model on a triangular lattice with ferromagnetic and antiferromagnetic interactions. Spin flips occur by means of either sequential or parallel updating with Glauber transition probabilities. They find a disagreement between the sequential and parallel updating schemes with respect to the calculation of thermodynamic properties from damage spreading. Their reasoning is as follows: For sequential Glauber, in the ferromagnetic case, they find the damage spreading transition temperature T_s is equal to the critical temperature T_c. In the antiferromagnetic case of sequential Glauber, which has no thermal transition, they find no spreading transition. When the same correspondence between T_c and T_s is not found to be true for parallel Glauber, they conclude that parallel Glauber dynamics is inadequate for measuring thermodynamic quantities. Though we agree with the conclusion that the parallel updating scheme should be used with great caution, their argument based on damage spreading is incorrect.

Parallel Glauber and sequential Glauber are two entirely different dynamics: Parallel Glauber can be thought of as sequential Glauber with the update being performed using the states of the neighbors at the previous time step exclusively. This implies that the definition of the energy of a spin differs in a subtle but significant way between sequential and parallel Glauber, and therefore the transition probabilities, and hence the dynamics, are different. When comparing different dynamics, it cannot be assumed that dynamical behavior, such as the damage spreading transition, will be of the same form.

While suggesting that a controversy exists regarding the equivalence of T_s and T_c, the authors' conclusions depend upon this equivalence. There is, in fact, no controversy: T_s is *not* in general equal to T_c. For example, for an Ising model on a 2D square lattice with Kawasaki (spin-exchange) transition probabilities, damage spreads at *all* temperatures; that is, $T_s \neq T_c$ for Kawasaki dynamics [2]. However, we know that Kawasaki dynamics is valid for simulating an Ising model. Therefore, whether or not T_s is equal to T_c does not determine the validity of a particular dynamics.

Furthermore, the damage spreading quantity that the authors measure to get T_s, the average damage sum or Hamming distance, has not been rigorously shown to relate to any thermal property of the system *except* in the special case of a ferromagnetic Ising model with heat-bath transition probabilities. If one wishes to measure a damage spreading quantity with a transition that is known to be related to the thermal transition, then one can measure a quantity known as the damage difference. There are two types of damage possible at a site: spin-up in system A and spin-down in system B, and the opposite. If the probabilities for finding each type of damage at a particular site are respectively denoted as d^{+-} and d^{-+}, then the average damage difference per spin is $d^{+-} - d^{-+}$, while the average Hamming distance, or damage sum, is $d^{+-} + d^{-+}$. It has been shown rigorously [3,4] that both the static and the time-dependent spin-spin correlation functions are proportional to the damage difference for any binary spin system and for all types of transition probabilities (Glauber, heat-bath, Kawasaki, etc.). Thus, generally, the damage difference is related to thermal properties while the damage sum is not, except in the fortuitous case of heat-bath transition probabilities, where $d^{-+}=0$ for ferromagnetic systems.

We emphasize that damage spreading provides a sensitive probe of the propagation of correlation in a binary spin system when the appropriate damage properties are evaluated. We support the authors' main conclusion that the parallel updating scheme should be rejected, but only on the basis of the nonergodicity that they demonstrate with this dynamics, which has been previously observed [5]. It is their rejection of a dynamics on the basis of the behavior of the Hamming distance (damage sum) with which we take issue.

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