Calculus 112 Practice Problems

Section 10.1 Problems #11, #15, #16

11. Let $f(x) = \sqrt{1-x} = (1-x)^{1/2}$. Then $f'(x) = -\frac{1}{2}(1-x)^{-1/2}$, $f''(x) = -\frac{1}{4}(1-x)^{-3/2}$, $f'''(x) = -\frac{3}{8}(1-x)^{-5/2}$. So $f(0) = 1$, $f'(0) = -\frac{1}{2}$, $f''(0) = -\frac{1}{4}$, $f'''(0) = -\frac{3}{8}$, and

$$\begin{aligned} P_3(x) &= 1 - \frac{1}{2}x - \frac{1}{4}\frac{1}{2!}x^2 - \frac{3}{8}\frac{1}{3!}x^3 \\ &= 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16}. \end{aligned}$$

15. Let $f(x) = \sin x$.
Then $f'(x) = \cos x$, $f''(x) = -\sin x$, and $f'''(x) = -\cos x$, so the Taylor polynomial for $\sin x$ of degree three about $x = -\pi/4$ is

$$\begin{aligned} P_3(x) &= \sin\left(-\frac{\pi}{4}\right) + \cos\left(-\frac{\pi}{4}\right)\left(x + \frac{\pi}{4}\right) \\ &\quad + \frac{-\sin\left(-\frac{\pi}{4}\right)}{2!}\left(x + \frac{\pi}{4}\right)^2 + \frac{-\cos\left(-\frac{\pi}{4}\right)}{3!}\left(x + \frac{\pi}{4}\right)^3 \\ &= \frac{\sqrt{2}}{2}\left(-1 + \left(x + \frac{\pi}{4}\right) + \frac{1}{2}\left(x + \frac{\pi}{4}\right)^2 - \frac{1}{6}\left(x + \frac{\pi}{4}\right)^3\right). \end{aligned}$$

16. Let $f(x) = \ln(x^2)$. Then $\ln(1^2) = \ln 1 = 0$.
Then $f'(x) = 2x^{-1}$, $f''(x) = -2x^{-2}$, $f'''(x) = 4x^{-3}$, and $f^{(4)}(x) = -12x^{-4}$.
The Taylor polynomial of degree 4 for $f(x) = \ln(x^2)$ about $x = 1$ is

$$\begin{aligned} P_4(x) &= \ln(1^2) + 2 \cdot 1^{-1}(x-1) + \frac{-2 \cdot 1^{-2}}{2!}(x-1)^2 + \frac{4 \cdot 1^{-3}}{3!}(x-1)^3 + \frac{-12 \cdot 1^{-4}}{4!}(x-1)^4 \\ &= 0 + 2(x-1) - (x-1)^2 + \frac{4}{6}(x-1)^3 - \frac{12}{24}(x-1)^4 \\ &= 2(x-1) - (x-1)^2 + \frac{2}{3}(x-1)^3 - \frac{1}{2}(x-1)^4. \end{aligned}$$