Calculus 112 Practice Problems

Section 10.3 Problems #1, #3, #7, #11

1. Substitute $y = -x$ into $e^y = 1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \cdots$. We get

$$\begin{aligned} e^{-x} &= 1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \cdots \\ &= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots. \end{aligned}$$

3. Substitute $x = \theta^2$ into series for $\cos x$:

$$\begin{aligned} \cos(\theta^2) &= 1 - \frac{(\theta^2)^2}{2!} + \frac{(\theta^2)^4}{4!} - \frac{(\theta^2)^6}{6!} + \cdots \\ &= 1 - \frac{\theta^4}{2!} + \frac{\theta^8}{4!} - \frac{\theta^{12}}{6!} + \cdots. \end{aligned}$$

7. Substituting $x = -z^2$ into $\frac{1}{\sqrt{1+x}} = (1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \cdots$ gives

$$\begin{aligned} \frac{1}{\sqrt{1-z^2}} &= 1 - \frac{(-z^2)}{2} + \frac{3(-z^2)^2}{8} - \frac{5(-z^2)^3}{16} + \cdots \\ &= 1 + \frac{1}{2}z^2 + \frac{3}{8}z^4 + \frac{5}{16}z^6 + \cdots. \end{aligned}$$

11.

$$\sqrt{(1+t)}\sin t = \left(1 + \frac{t}{2} - \frac{t^2}{8} + \frac{t^3}{16} - \cdots\right)\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \cdots\right)$$

Multiplying and collecting terms yields

$$\begin{aligned} \sqrt{(1+t)}\sin t &= t + \frac{t^2}{2} - \left(\frac{t^3}{3!} + \frac{t^3}{8}\right) + \left(\frac{t^4}{16} - \frac{t^4}{12}\right) + \cdots \\ &= t + \frac{1}{2}t^2 - \frac{7}{24}t^3 - \frac{1}{48}t^4 + \cdots. \end{aligned}$$