Calculus 112 Practice Problems

Section 11.2 Problems #12, #13, #14

12. (a) Since $y' = -y$, the slope is negative above the x-axis (when y is positive) and positive below the x-axis (when y is negative). The only slope field for which this is true is II.

(b) Since $y' = y$, the slope is positive for positive y and negative for negative y. This is true of both I and III. As y get larger, the slope should get larger, so the correct slope field is I.

(c) Since $y' = x$, the slope is positive for positive x and negative for negative x. This corresponds to slope field V.

(d) Since $y' = \dfrac{1}{y}$, the slope is positive for positive y and negative for negative y. As y approaches 0, the slope becomes larger in magnitude, which correspond to solution curves close to vertical. The correct slope field is III.

(e) Since $y' = y^2$, the slope is always positive, so this must correspond to slope field IV.

13. (a) II (b) VI (c) IV (d) I (e) III (f) V

14. (a) We know that when $x = 0$, $\dfrac{dy}{dx} = e^{x^2} = e^0 = 1$.

We know that when $x = 1$, $\dfrac{dy}{dx} = e^{x^2} = e^1 = e$.

We know that when $x = 2$, $\dfrac{dy}{dx} = e^{x^2} = e^4 \approx 54.6$.

Thus the appropriate slope field is (IV).

(b) We know that when $x = 0$, $\dfrac{dy}{dx} = e^{-2x^2} = e^0 = 1$.

We know that when $x = 1$, $\dfrac{dy}{dx} = e^{-2x^2} = e^{-2} \approx 0.14$.

We know that when $x = 2$, $\dfrac{dy}{dx} = e^{-2x^2} = e^{-8} \approx 0.000335$.

Thus the appropriate slope field is (I).

(c) We know that when $x = 0$, $\dfrac{dy}{dx} = e^{-x^2/2} = e^0 = 1$.

We know that when $x = 1$, $\dfrac{dy}{dx} = e^{-x^2/2} = e^{-1/2} \approx 0.61$.

We know that when $x = 2$, $\dfrac{dy}{dx} = e^{-x^2/2} = e^{-2} \approx 0.14$.

Thus the appropriate slope field is (III).

(d) We know that when $x = 0$, $\dfrac{dy}{dx} = e^{-0.5x}\cos x = e^0 \cos 0 = 1$.

We know that when $x = 1$, $\dfrac{dy}{dx} = e^{-0.5x}\cos x = e^{-1/2}\cos 1 \approx 0.33$.

We know that when $x = 2$, $\dfrac{dy}{dx} = e^{-0.5x}\cos x = e^{-1}\cos 2 \approx -0.153$.

Thus the appropriate slope field is (V).

(e) We know that when $x = 0$, $\dfrac{dy}{dx} = \dfrac{1}{(1 + 0.5\cos x)^2} \approx 0.44$.

We know that when $x = 1$, $\dfrac{dy}{dx} = \dfrac{1}{(1 + 0.5\cos x)^2} \approx 0.61$.

We know that when $x = 2$, $\dfrac{dy}{dx} = \dfrac{1}{(1 + 0.5\cos x)^2} \approx 1.59$.

Thus the appropriate slope field is (II).

(f) We know that when $x = 0$, $\dfrac{dy}{dx} = -e^{-x^2} = -1$.

We know that when $x = 1$, $\dfrac{dy}{dx} = -e^{-x^2} \approx -0.37$.

We know that when $x = 2$, $\dfrac{dy}{dx} = -e^{-x^2} \approx -0.14$.

Thus the appropriate slope field is (VI).