Calculus 112 Practice Problems

Section 11.4 Problems #2, #13, #17, #23

2. Separating variables gives

$$\int \frac{dP}{P} = \int 0.02\,dt,$$

so

$$\ln|P| = 0.02t + C.$$

Thus

$$|P| = e^{0.02t+C}$$

and

$$P = Ae^{0.02t}, \text{ where } A = \pm e^C.$$

We are given $P(0) = 20$. Therefore, $P(0) = Ae^{(0.02)\cdot 0} = A = 20$. So the solution is

$$P = 20e^{0.02t}.$$

13. Separating variables gives

$$\int \frac{dy}{y-200} = \int 0.5dt$$

$$\ln|y-200| = 0.5t + C$$

$$y = 200 + Ae^{0.5t}, \quad \text{where } A = \pm e^C.$$

The initial condition, $y(0) = 50$, gives

$$50 = 200 + A, \quad \text{so} \quad A = -150.$$

Thus,

$$y = 200 - 150e^{0.5t}.$$

17. Factoring out the 0.1 gives

$$\frac{dm}{dt} = 0.1m + 200 = 0.1(m + 2000)$$

$$\int \frac{dm}{m+2000} = \int 0.1\,dt,$$

so

$$\ln|m + 2000| = 0.1t + C,$$

and

$$m = Ae^{0.1t} - 2000, \text{ where } A = \pm e^C.$$

Using the initial condition, $m(0) = Ae^{(0.1)\cdot 0} - 2000 = 1000$, gives $A = 3000$. Thus

$$m = 3000e^{0.1t} - 2000.$$

23. Separating variables gives

$$\frac{dy}{dt} = y^2(1+t)$$
$$\int \frac{dy}{y^2} = \int (1+t)\,dt,$$

so

$$-\frac{1}{y} = t + \frac{t^2}{2} + C,$$

giving

$$y = -\frac{1}{t + t^2/2 + C}.$$

Since $y = 2$ when $t = 1$, we have

$$2 = -\frac{1}{1 + 1/2 + C}, \quad \text{so} \quad 2C + 3 = -1, \quad \text{and} \quad C = -2.$$

Thus

$$y = -\frac{1}{t^2/2 + t - 2} = -\frac{2}{t^2 + 2t - 4}.$$