Calculus 112 Practice Problems

Section 11.5 Problems 13, #15, #17

13. (a) Since we are told that the rate at which the quantity of the drug decreases is proportional to the amount of the drug left in the body, we know the differential equation modeling this situation is

$$\frac{dQ}{dt} = kQ.$$

Since we are told that the quantity of the drug is decreasing, we know that $k < 0$.

(b) We know that the general solution to the differential equation

$$\frac{dQ}{dt} = kQ$$

is

$$Q = Ce^{kt}.$$

(c) We are told that the half life of the drug is 3.8 hours. This means that at $t = 3.8$, the amount of the drug in the body is half the amount that was in the body at $t = 0$, or, in other words,

$$0.5Q(0) = Q(3.8).$$

Solving this equation gives

$$\begin{aligned} 0.5Q(0) &= Q(3.8) \\ 0.5Ce^{k(0)} &= Ce^{k(3.8)} \\ 0.5C &= Ce^{k(3.8)} \\ 0.5 &= e^{k(3.8)} \\ \ln(0.5) &= k(3.8) \\ \frac{\ln(0.5)}{3.8} &= k \\ k &\approx -0.182. \end{aligned}$$

(d) From part (c) we know that the formula for Q is

$$Q = Ce^{-0.182t}.$$

We are told that initially there are 10 mg of the drug in the body. Thus at $t = 0$, we get

$$10 = Ce^{-0.182(0)}$$

so

$$C = 10.$$

Thus our equation becomes

$$Q(t) = 10e^{-0.182t}.$$

Substituting $t = 12$, we get

$$\begin{aligned} Q(t) &= 10e^{-0.182t} \\ Q(12) &= 10e^{-0.182(12)} \\ &= 10e^{-2.184} \\ Q(12) &\approx 1.126 \text{ mg}. \end{aligned}$$

15. (a) Letting k be the constant of proportionality, by Newton's Law of Cooling, we have

$$\frac{dH}{dt} = k(68 - H).$$

(b) We solve this equation by separating variables:

$$\begin{aligned}
\int \frac{dH}{68 - H} &= \int k\, dt \\
-\ln|68 - H| &= kt + C \\
68 - H &= \pm e^{C - kt} \\
H &= 68 - Ae^{-kt}.
\end{aligned}$$

(c) We are told that $H = 40$ when $t = 0$; this tells us that

$$\begin{aligned}
40 &= 68 - Ae^{-k(0)} \\
40 &= 68 - A \\
A &= 28.
\end{aligned}$$

Knowing A, we can solve for k using the fact that $H = 48$ when $t = 1$:

$$\begin{aligned}
48 &= 68 - 28e^{-k(1)} \\
\frac{20}{28} &= e^{-k} \\
k &= -\ln\left(\frac{20}{28}\right) = 0.33647.
\end{aligned}$$

So the formula is $H(t) = 68 - 28e^{-0.33647t}$. We calculate H when $t = 3$, by

$$H(3) = 68 - 28e^{-0.33647(3)} = 57.8^\circ\text{F}.$$

17. According to Newton's Law of Cooling, the temperature, T, of the roast as a function of time, t, satisfies

$$\begin{aligned} T'(t) &= k(350 - T) \\ T(0) &= 40. \end{aligned}$$

Solving this differential equation, we get that $T = 350 - 310e^{-kt}$ for some $k > 0$. To find k, we note that at $t = 1$ we have $T = 90$, so

$$\begin{aligned} 90 &= 350 - 310e^{-k(1)} \\ \frac{260}{310} &= e^{-k} \\ k &= -\ln\left(\frac{260}{310}\right) \\ &\approx 0.17589. \end{aligned}$$

Thus, $T = 350 - 310e^{-0.17589t}$. Solving for t when $T = 140$, we have

$$\begin{aligned} 140 &= 350 - 310e^{-0.17589t} \\ \frac{210}{310} &= e^{-0.17589t} \\ t &= \frac{\ln(210/310)}{-0.17589} \\ t &\approx 2.21 \text{ hours}. \end{aligned}$$