Calculus 112 Practice Problems

Section 11.6 Problems #9, #11, #15

9. Let $C(t)$ be the current flowing in the circuit at time t, then

$$\frac{dC}{dt} = -\alpha C$$

where $\alpha > 0$ is the constant of proportionality between the rate at which the current decays and the current itself.

The general solution of this differential equation is $C(t) = Ae^{-\alpha t}$ but since $C(0) = 30$, we have that $A = 30$, and so we get the particular solution $C(t) = 30e^{-\alpha t}$.

When $t = 0.01$, the current has decayed to 11 amps so that $11 = 30e^{-\alpha 0.01}$ which gives $\alpha = -100\ln(11/30) = 100.33$ so that,

$$C(t) = 30e^{-100.33t}.$$

11. (a) If I is intensity and l is the distance traveled through the water, then for some $k > 0$,

$$\frac{dI}{dl} = -kI.$$

(The proportionality constant is negative because intensity decreases with distance). Thus $I = Ae^{-kl}$. Since $I = A$ when $l = 0$, A represents the initial intensity of the light.

(b) If 50% of the light is absorbed in 10 feet, then $0.50A = Ae^{-10k}$, so $e^{-10k} = \frac{1}{2}$, giving

$$k = \frac{-\ln\frac{1}{2}}{10} = \frac{\ln 2}{10}.$$

In 20 feet, the percentage of light left is

$$e^{-\frac{\ln 2}{10}\cdot 20} = e^{-2\ln 2} = (e^{\ln 2})^{-2} = 2^{-2} = \frac{1}{4},$$

so $\frac{3}{4}$ or 75% of the light has been absorbed. Similarly, after 25 feet,

$$e^{-\frac{\ln 2}{10}\cdot 25} = e^{-2.5\ln 2} = (e^{\ln 2})^{-\frac{5}{2}} = 2^{-\frac{5}{2}} \approx 0.177.$$

Approximately 17.7% of the light is left, so 82.3% of the light has been absorbed.

15. (a) Since the rate of change of the weight is equal to

$$\frac{1}{3500}(\text{Intake} \ - \ \text{Amount to maintain weight})$$

we have

$$\frac{dW}{dt} = \frac{1}{3500}(I - 20W).$$

(b) Starting off with the equation

$$\frac{dW}{dt} = -\frac{1}{175}\left(W - \frac{I}{20}\right),$$

we separate variables and integrate:

$$\int \frac{dW}{W - \frac{I}{20}} = -\int \frac{1}{175}\,dt.$$

Thus we have

$$\ln\left|W - \frac{I}{20}\right| = -\frac{1}{175}t + C$$

so that

$$W - \frac{I}{20} = Ae^{-\frac{1}{175}t}$$

or in other words

$$W = \frac{I}{20} + Ae^{-\frac{1}{175}t}.$$

Let us call the person's initial weight W_0 at $t = 0$. Then $W_0 = \frac{I}{20} + Ce^0$, so $C = W_0 - \frac{I}{20}$. Thus

$$W = \frac{I}{20} + \left(W_0 - \frac{I}{20}\right)e^{-\frac{1}{175}t}.$$

(c) Using part (b), we have $W = 150 + 10e^{-\frac{1}{175}t}$. This means that $W \to 150$ as $t \to \infty$. See Figure 11.30.

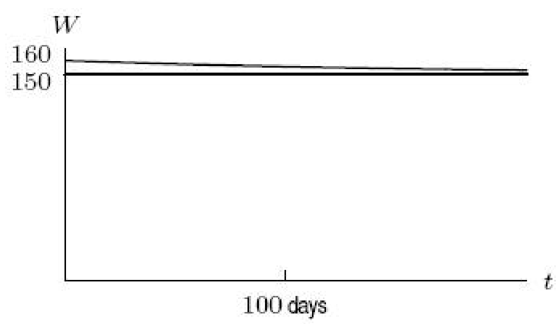

Figure 11.30