Calculus 112 Practice Problems

Section 11.7 Problems #21, #23

21. By rewriting the equation, we see that it is logistic:

$$\frac{1}{P}\frac{dP}{dt} = \frac{(100 - P)}{1000}.$$

Before looking at its solution, we explain why there must always be at least 100 individuals. Since the population begins at 200, the quantity dP/dt is initially negative, so the population initially decreases. It continues to do so while $P > 100$. If the population ever reached 100, then dP/dt would be 0. This would mean the population stopped changing—so if the population ever decreased to 100, that's where it would stay. The fact that dP/dt is always negative for $P > 100$ also shows that the population is always under 200, as shown in Figure 11.36.

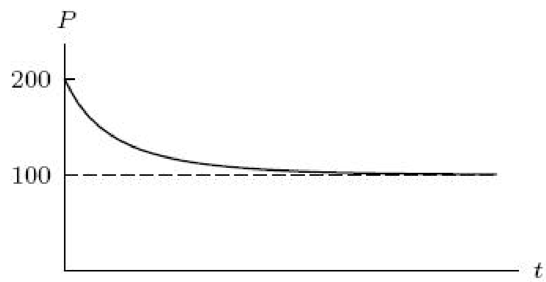

Figure 11.36

The solution, as given by the formula derived in the chapter, is

$$P = \frac{100}{1 - 0.5e^{-0.1t}}$$

23. (a) We know that a logistic curve can be modeled by the function

$$P = \frac{L}{1 + Ae^{-kt}}$$

where $A = (L - P_0)/(P_0)$ and P is the number of people infected by the virus at a particular time t. We know that L is the limiting value, or the maximal number of people infected with the virus, so in our case

$$L = 5000.$$

We are also told that initially there are only ten people infected with the virus so that we get

$$P_0 = 10.$$

Thus we have

$$\begin{aligned} A &= \frac{L - P_0}{P_0} \\ &= \frac{5000 - 10}{10} \\ &= 499. \end{aligned}$$

We are also told that in the early stages of the virus, infection grows exponentially with $k = 1.78$. Thus we get that the logistic function for people infected is

$$P = \frac{5000}{1 + 499e^{-1.78t}}.$$

(b) See Figure 11.37.

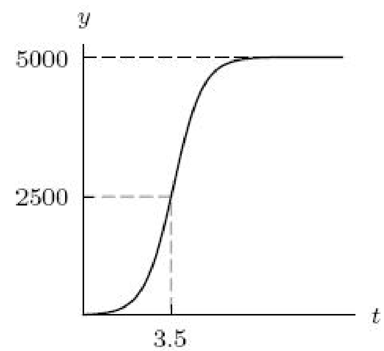

Figure 11.37

(c) Looking at the graph we see that the the point at which the rate changes from increasing to decreasing, the inflection point, occurs at roughly $t = 3.5$ giving a value of $P = 2500$. Thus after roughly 2500 people have been infected, the rate of infection starts dropping. See above.