Calculus 112 Practice Problems

Section 5.1 Problems #1, #3, #9, #27

1. (a) (i) Since the velocity is increasing, for an upper estimate we use a right sum. Using $n = 4$, we have $\Delta t = 3$, so

$$\text{Upper estimate } = (37)(3) + (38)(3) + (40)(3) + (45)(3) = 480.$$

(ii) Using $n = 2$, we have $\Delta t = 6$, so

$$\text{Upper estimate } = (38)(6) + (45)(6) = 498.$$

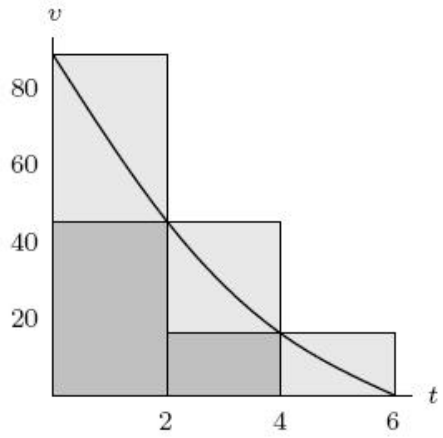
(b) The answer using $n = 4$ is more accurate as it uses the values of $v(t)$ when $t = 3$ and $t = 9$.

(c) Since the velocity is increasing, for a lower estimate we use a left sum. Using $n = 4$, we have $\Delta t = 3$, so

$$\text{Lower estimate } = (34)(3) + (37)(3) + (38)(3) + (40)(3) = 447.$$

3. (a) Lower estimate $= (45)(2) + (16)(2) + (0)(2) = 122$ feet.
Upper estimate $= (88)(2) + (45)(2) + (16)(2) = 298$ feet.

(b)



9. Using $\Delta t = 0.2$, our upper estimate is

$$\frac{1}{1+0}(0.2) + \frac{1}{1+0.2}(0.2) + \frac{1}{1+0.4}(0.2) + \frac{1}{1+0.6}(0.2) + \frac{1}{1+0.8}(0.2) \approx 0.75.$$

The lower estimate is

$$\frac{1}{1+0.2}(0.2) + \frac{1}{1+0.4}(0.2) + \frac{1}{1+0.6}(0.2) + \frac{1}{1+0.8}(0.2)\frac{1}{1+1}(0.2) \approx 0.65.$$

Since v is a decreasing function, the bug has crawled more than 0.65 meters, but less than 0.75 meters. We average the two to get a better estimate:

$$\frac{0.65 + 0.75}{2} = 0.70 \text{ meters.}$$

27. (a) Since car B starts at $t = 2$, the tick marks on the horizontal axis (which we assume are equally spaced) are 2 hours apart. Thus car B stops at $t = 6$ and travels for 4 hours.
Car A starts at $t = 0$ and stops at $t = 8$, so it travels for 8 hours.

(b) Car A's maximum velocity is approximately twice that of car B, that is 100 km/hr.

(c) The distance traveled is given by the area of under the velocity graph. Using the formula for the area of a triangle, the distances are given approximately by

$$\text{Car } A \text{ travels} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 8 \cdot 100 = 400 \text{ km}$$

$$\text{Car } B \text{ travels} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 4 \cdot 50 = 100 \text{ km}.$$