Calculus 112 Practice Problems

Section 5.2 Problems #4-8, #15, #17, #27, #35

4. We know that

$$\int_{-10}^{15} f(x)dx = \text{Area under } f(x) \text{ between } x = -10 \text{ and } x = 15.$$

The area under the curve consists of approximately 14 boxes, and each box has area $(5)(5) = 25$. Thus, the area under the curve is about $(14)(25) = 350$, so

$$\int_{-10}^{15} f(x)dx \approx 350.$$

5. With $\Delta x = 5$, we have

$$\text{Left-hand sum } = 5(0 + 100 + 200 + 100 + 200 + 250 + 275) = 5625,$$

$$\text{Right-hand sum } = 5(100 + 200 + 100 + 200 + 250 + 275 + 300) = 7125.$$

The average of these two sums is our best guess for the value of the integral;

$$\int_{-15}^{20} f(x)\,dx \approx \frac{5625 + 7125}{2} = 6375.$$

6. The graph given shows that f is positive for $0 \le t \le 1$. Since the graph is contained within a rectangle of height 100 and length 1, the answers -98.35 and 100.12 are both either too small or too large to represent $\int_0^1 f(t)dt$. Since the graph of f is above the horizontal line $y = 80$ for $0 \le t \le 0.95$, the best estimate is 93.47 and not 71.84.

7. We estimate $\int_0^{40} f(x)dx$ using left- and right-hand sums:

$$\text{Left sum} = 350 \cdot 10 + 410 \cdot 10 + 435 \cdot 10 + 450 \cdot 10 = 16{,}450.$$

$$\text{Right sum} = 410 \cdot 10 + 435 \cdot 10 + 450 \cdot 10 + 460 \cdot 10 = 17{,}550.$$

We estimate that

$$\int_0^{40} f(x)dx \approx \frac{16450 + 17550}{2} = 17{,}000.$$

In this estimate, we used $n = 4$ and $\Delta x = 10$.

8. We take $\Delta x = 3$. Then:

$$\begin{aligned}
\text{Left-hand sum} &= 50(3) + 48(3) + 44(3) + 36(3) + 24(3) \\
&= 606 \\
\text{Right-hand sum} &= 48(3) + 44(3) + 36(3) + 24(3) + 8(3) \\
&= 480 \\
\text{Average} &= \frac{606 + 480}{2} = 543.
\end{aligned}$$

So,

$$\int_0^{15} f(x)\,dx \approx 543.$$

15. Since $\cos t \geq 0$ for $0 \leq t \leq \pi/2$, the area is given by

$$\text{Area} = \int_0^{\pi/2} \cos t\, dt = 1.$$

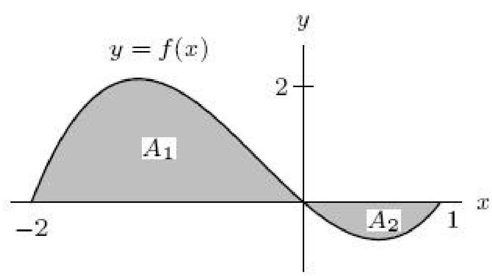
The integral was evaluated on a calculator.

17. A graph of $y = \ln x$ shows that this function is non-negative on the interval $x = 1$ to $x = 4$. Thus,

$$\text{Area} = \int_1^4 \ln x\, dx = 2.545.$$

The integral was evaluated on a calculator.

27. (a)



(b) $\text{A}_1 = \int_{-2}^{0} f(x)\, dx = 2.667.$

$\text{A}_2 = -\int_0^1 f(x)\, dx = 0.417.$

So total area $= A_1 + A_2 \approx 3.084$. Note that while A_1 and A_2 are accurate to 3 decimal places, the quoted value for $A_1 + A_2$ is accurate only to 2 decimal places.

(c) $\int_{-2}^{1} f(x)\, dx = A_1 - A_2 = 2.250.$

35. See Figure 5.26.

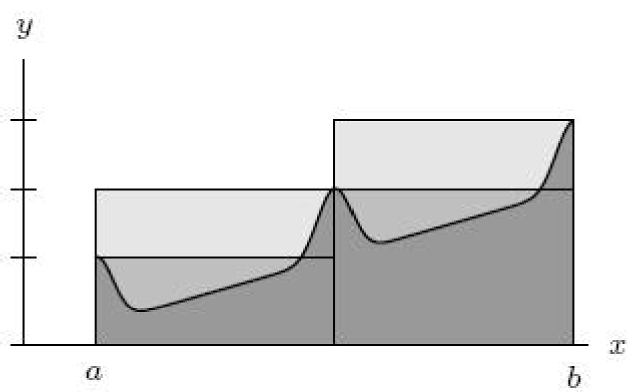


Figure 5.26: Integral vs. Left- and Right-Hand Sums