Calculus 112 Practice Problems

Section 6.1 Problems #5, #15, #17, #24

5. Since dP/dt is negative for $t < 3$ and positive for $t > 3$, we know that P is decreasing for $t < 3$ and increasing for $t > 3$. Between each two integer values, the magnitude of the change is equal to the area between the graph dP/dt and the t-axis. For example, between $t = 0$ and $t = 1$, we see that the change in P is -1. Since $P = 2$ at $t = 0$, we must have $P = 1$ at $t = 1$. The other values are found similarly, and are shown in Table 6.1.

Table 6.1

t	1	2	3	4	5
P	1	0	$-1/2$	0	1

15. We can start by finding four points on the graph of $F(x)$. The first one is given: $F(2) = 3$. By the Fundamental Theorem of Calculus, $F(6) = F(2) + \int_2^6 F'(x)dx$. The value of this integral is -7 (the area is 7, but the graph lies below the x-axis), so $F(6) = 3 - 7 = -4$. Similarly, $F(0) = F(2) - 2 = 1$, and $F(8) = F(6) + 4 = 0$. We sketch a graph of $F(x)$ by connecting these points, as shown in Figure 6.10.

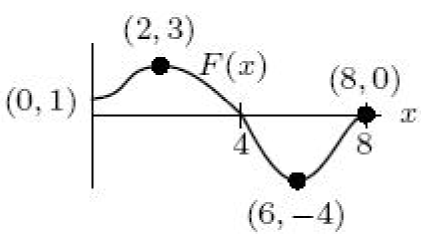


Figure 6.10

17. Looking at the graph of g' in Figure 6.12, we see that the critical points of g occur when $x = 15$ and $x = 40$, since $g'(x) = 0$ at these values. Inflection points of g occur when $x = 10$ and $x = 20$, because $g'(x)$ has a local maximum or minimum at these values. Knowing these four key points, we sketch the graph of $g(x)$ in Figure 6.13.

We start at $x = 0$, where $g(0) = 50$. Since g' is negative on the interval $[0, 10]$, the value of $g(x)$ is decreasing there. At $x = 10$ we have

$$\begin{aligned} g(10) &= g(0) + \int_0^{10} g'(x)\,dx \\ &= 50 - (\text{area of shaded trapezoid } T_1) \\ &= 50 - \left(\frac{10+20}{2} \cdot 10\right) = -100. \end{aligned}$$

Similarly,

$$\begin{aligned} g(15) &= g(10) + \int_{10}^{15} g'(x)\,dx \\ &= -100 - (\text{area of triangle } T_2) \\ &= -100 - \frac{1}{2}(5)(20) = -150. \end{aligned}$$

Continuing,

$$g(20) = g(15) + \int_{15}^{20} g'(x)\,dx = -150 + \frac{1}{2}(5)(10) = -125,$$

and

$$g(40) = g(20) + \int_{20}^{40} g'(x)\,dx = -125 + \frac{1}{2}(20)(10) = -25.$$

We now find concavity of $g(x)$ in the intervals $[0, 10]$, $[10, 15]$, $[15, 20]$, $[20, 40]$ by checking whether $g'(x)$ increases or decreases in these same intervals. If $g'(x)$ increases, then $g(x)$ is concave up; if $g'(x)$ decreases, then $g(x)$ is concave down. Thus we finally have the graph of $g(x)$ in Figure 6.13.

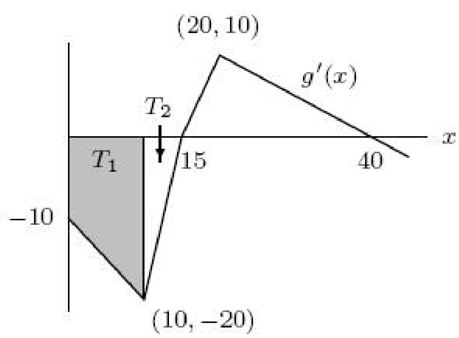


Figure 6.12

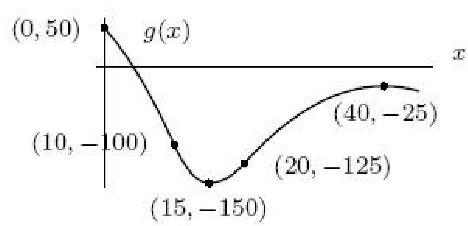
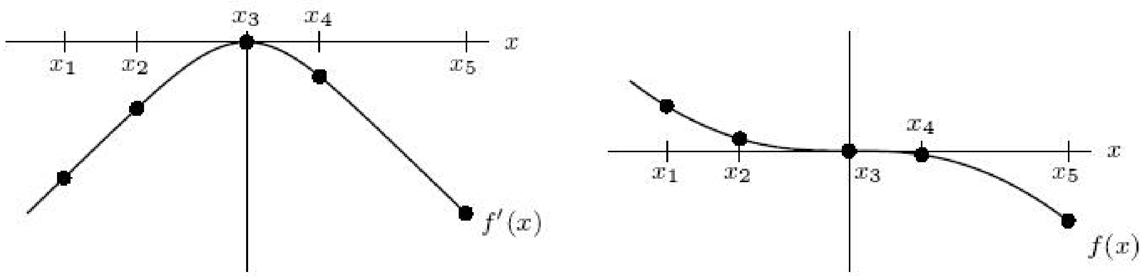


Figure 6.13

24.



(a) $f(x)$ is greatest at x_1.
(b) $f(x)$ is least at x_5.
(c) $f'(x)$ is greatest at x_3..
(d) $f'(x)$ is least at x_5.
(e) $f''(x)$ is greatest at x_1.
(f) $f''(x)$ is least at x_5.