Calculus 112 Practice Problems

Section 6.2 Problems #37-52, #69, #73, #75, #78

37. $\int 5x\,dx = \frac{5}{2}x^2 + C.$

38. $\int x^3\,dx = \frac{x^4}{4} + C$

39. $\int \sin\theta\,d\theta = -\cos\theta + C$

40. $\int (x^3 - 2)\,dx = \frac{x^4}{4} - 2x + C$

41. $\int (t^2 + \frac{1}{t^2})\,dt = \frac{t^3}{3} - \frac{1}{t} + C$

42. $\int 4\sqrt{w}\,dw = \frac{8}{3}w^{3/2} + C$

43. $\int (x^2 + 5x + 8)\,dx = \frac{x^3}{3} + \frac{5x^2}{2} + 8x + C$

44. $\int \frac{4}{t^2}\,dt = -\frac{4}{t} + C$

45. $2t^2 + 7t + C$

46. $\sin\theta + C$

47. $5e^z + C$

48. $\frac{x^2}{2} + 2x^{1/2} + C$

49. $-\cos t + C$

50. $\pi x + \frac{x^{12}}{12} + C$

51. $\int \left(t^{3/2} + t^{-3/2}\right)\,dt = \frac{2t^{5/2}}{5} - 2t^{-1/2} + C$

52. $\int \left(y - \frac{1}{y}\right)^2\,dy = \int \left(y^2 - 2 + \frac{1}{y^2}\right)\,dy = \frac{y^3}{3} - 2y - \frac{1}{y} + C$

69. The graph is shown in Figure 6.23. Since $\cos\theta \geq \sin\theta$ for $0 \leq \theta \leq \pi/4$, we have

$$\begin{aligned}\text{Area} &= \int_0^{\pi/4} (\cos\theta - \sin\theta)\, d\theta \\ &= (\sin\theta + \cos\theta)\Big|_0^{\pi/4} \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 = \sqrt{2} - 1.\end{aligned}$$

73. The area under $f(x) = 8x$ between $x = 1$ and $x = b$ is given by $\int_1^b (8x)dx$. Using the Fundamental Theorem to evaluate the integral:

$$\text{Area} = 4x^2\Big|_1^b = 4b^2 - 4.$$

Since the area is 192, we have

$$\begin{aligned}4b^2 - 4 &= 192 \\ 4b^2 &= 196 \\ b^2 &= 49 \\ b &= \pm 7.\end{aligned}$$

Since b is larger than 1, we have $b = 7$.

75. We have

$$\text{Average value} = \frac{1}{10-0}\int_0^{10} (x^2+1)dx = \frac{1}{10}\left(\frac{x^3}{3} + x\right)\Big|_0^{10} = \frac{1}{10}\left(\frac{10^3}{3} + 10 - 0\right) = \frac{103}{3}.$$

We see in Figure 6.27 that the average value of $103/3 \approx 34.33$ for $f(x)$ looks right.