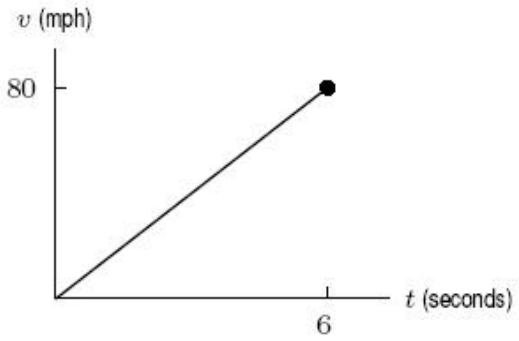
Calculus 112 Practice Problems

Section 6.3 Problems #18, #23

18. Since the car's acceleration is constant, a graph of its velocity against time t is linear, as shown below.



The acceleration is just the slope of this line:

$$\frac{dv}{dt} = \frac{80 - 0 \text{ mph}}{6 \text{ sec}} = \frac{40}{3} = 13.33 \frac{\text{mph}}{\text{sec}}.$$

To convert our units into ft/sec^2,

$$\frac{40}{3} \cdot \frac{\text{mph}}{\text{sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \frac{1 \text{ hour}}{3600 \text{ sec}} = 19.55 \frac{\text{ft}}{\text{sec}^2}$$

23. (a) The velocity is decreasing at 32 ft/sec^2, the acceleration due to gravity.
(b) The graph is a line because the velocity is decreasing at a constant rate.
(c) The highest point is reached when the velocity is 0, which occurs when

$$\text{Time} = \frac{160}{32} = 5 \text{ sec.}$$

(d) The object hits the ground at $t = 10$ seconds, since by symmetry if the object takes 5 seconds to go up, it takes 5 seconds to come back down.
(e) See Figure 6.34.
(f) The maximum height is the distance traveled when going up, which is represented by the area A of the triangle above the time axis.

$$\text{Area} = \frac{1}{2}(160 \text{ ft/sec})(5 \text{ sec}) = 400 \text{ feet.}$$

(g) The slope of the line is -32 so

$$v(t) = -32t + 160.$$

Antidifferentiating, we get

$$s(t) = -16t^2 + 160t + s_0.$$

Since the object starts on the ground, $s_0 = 0$, so

$$s(t) = -16t^2 + 160t.$$

At $t = 5$, we have

$$s(t) = -400 + 800 = 400 \text{ ft}.$$

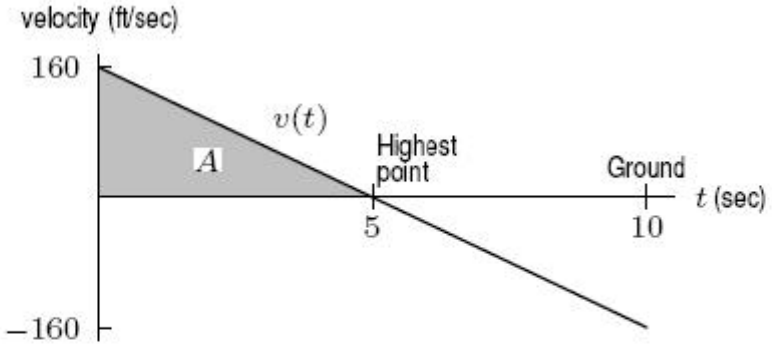


Figure 6.34