Calculus 112 Practice Problems

Section 6.4 Problems #29-36, #37

29. If we let $f(x) = \int_2^x \sin(t^2)\,dt$ and $g(x) = x^3$, using the chain rule gives

$$\frac{d}{dx}\int_2^{x^3} \sin(t^2)\,dt = f'(g(x)) \cdot g'(x) = \sin((x^3)^2) \cdot 3x^2 = 3x^2 \sin(x^6).$$

30. If we let $f(t) = \int_1^t \cos(x^2)\,dx$ and $g(t) = \sin t$, using the chain rule gives

$$\frac{d}{dt}\int_1^{\sin t} \cos(x^2)\,dx = f'(g(t)) \cdot g'(t) = \cos((\sin t)^2) \cdot \cos t = \cos(\sin^2 t)(\cos t).$$

31. If we let $F(x) = \int_0^x \ln(1+t^2)\,dt$, using the chain rule gives

$$\frac{d}{dx}F(x^2) = 2xF'(x^2) = 2x\ln(1+(x^2)^2) = 2x\ln(1+x^4).$$

32. We first write

$$\int_{2t}^{4} \sin(\sqrt{x})\,dx$$

as

$$-\int_4^{2t} \sin(\sqrt{x})\,dx.$$

Letting

$$F(t) = -\int_4^t \sin(\sqrt{x})\,dx,$$

and using the chain rule gives

$$\frac{d}{dt}F(2t) = 2F'(2t) = 2[-\sin(\sqrt{2t})] = -2\sin(\sqrt{2t}).$$

33. Since $\int_{\cos x}^3 e^{t^2}\,dt = -\int_3^{\cos x} e^{t^2}\,dt$, if we let $f(x) = \int_3^x e^{t^2}\,dt$ and $g(x) = \cos x$, using the chain rule gives

$$\frac{d}{dx}\int_{\cos x}^{3} e^{t^2}\,dt = -\frac{d}{dx}\int_3^{\cos x} e^{t^2}\,dt = -f'(g(x)) \cdot g'(x) = -e^{(\cos x)^2}(-\sin x) = \sin x e^{\cos^2 x}.$$

34. If we split the integral at $x = 0$, we have

$$\int_{-x}^{x} e^{-t^4}\,dt = \int_{-x}^{0} e^{-t^4}\,dt + \int_0^x e^{-t^4}\,dt = -\int_0^{-x} e^{-t^4}\,dt + \int_0^x e^{-t^4}\,dt.$$

If we let

$$F(x) = \int_0^x e^{-t^4}\,dt,$$

using the chain rule on each part separately gives

$$\frac{d}{dx}[-F(-x) + F(x)] = -(-1)F'(-x) + (1)F'(x) = e^{-(-x)^4} + e^{-x^4} = 2e^{-x^4}.$$

35. If we split the integral at $x = 0$, we have

$$\int_{-x^2}^{x^2} e^{t^2}\,dt = \int_{-x^2}^{0} e^{t^2}\,dt + \int_{0}^{x^2} e^{t^2}\,dt = -\int_{0}^{-x^2} e^{t^2}\,dt + \int_{0}^{x^2} e^{t^2}\,dt.$$

If we let

$$F(x) = \int_0^x e^{t^2}\,dt,$$

using the chain rule on each part separately gives

$$\frac{d}{dx}[-F(-x^2) + F(x^2)] = -(-2x)F'(-x^2) + (2x)F'(x^2) = (2x)e^{(-x^2)^2} + (2x)e^{(x^2)^2} = 4xe^{x^4}.$$

36. We split the integral at $x = 1$ (or any other point we choose):

$$\int_{e^t}^{t^3} \sqrt{1+x^2}\,dx = \int_1^{t^3} \sqrt{1+x^2}\,dx + \int_{e^t}^{1} \sqrt{1+x^2}\,dx = \int_1^{t^3} \sqrt{1+x^2}\,dx - \int_1^{e^t} \sqrt{1+x^2}\,dx.$$

Differentiating each part separately and using the chain rule gives

$$\begin{aligned}\frac{d}{dt}\int_{e^t}^{t^3} \sqrt{1+x^2}\,dx &= \frac{d}{dt}\int_1^{t^3} \sqrt{1+x^2}\,dx - \frac{d}{dt}\int_1^{e^t} \sqrt{1+x^2}\,dx \\ &= \sqrt{1+(t^3)^2}\cdot 3t^2 - \sqrt{1+(e^t)^2}\cdot e^t \\ &= 3t^2\sqrt{1+t^6} - e^t\sqrt{1+e^{2t}}.\end{aligned}$$

37. (a) The definition of P gives

$$P(0) = \int_0^0 \arctan(t^2)\,dt = 0$$

and

$$P(-x) = \int_0^{-x} \arctan(t^2)\,dt.$$

Changing the variable of integration by letting $t = -z$ gives

$$\int_0^{-x} \arctan(t^2)\,dt = \int_0^{x} \arctan((-z)^2)(-dz) = -\int_0^{x} \arctan(z^2)\,dz.$$

Thus P is an odd function.

(b) Using the Second Fundamental Theorem gives $P'(x) = \arctan(x^2)$, which is greater than 0 for $x \neq 0$. Thus P is increasing everywhere.

(c) Since

$$P''(x) = \frac{2x}{1+x^4},$$

we have P concave up if $x > 0$ and concave down if $x < 0$.

(d) See Figure 6.43.

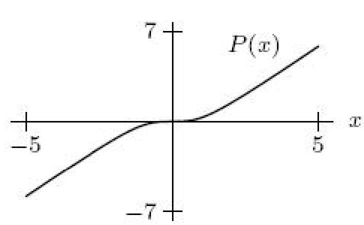

Figure 6.43