Calculus 112 Practice Problems

Section 7.1 Problems #3-46, #109

3. Make the substitution $w = t^2$, $dw = 2t\,dt$. The general antiderivative is $\int te^{t^2}\,dt = (1/2)e^{t^2} + C$.

4. We use the substitution $w = 3x$, $dw = 3\,dx$.

$$\int e^{3x}dx = \frac{1}{3}\int e^w dw = \frac{1}{3}e^w + C = \frac{1}{3}e^{3x} + C.$$

Check: $\frac{d}{dx}(\frac{1}{3}e^{3x} + C) = \frac{1}{3}e^{3x}(3) = e^{3x}$.

5. We use the substitution $w = -x$, $dw = -\,dx$.

$$\int e^{-x}dx = -\int e^w dw = -e^w + C = -e^{-x} + C.$$

Check: $\frac{d}{dx}(-e^{-x} + C) = -(-e^{-x}) = e^{-x}$.

6. We use the substitution $w = -0.2t$, $dw = -0.2\,dt$.

$$\int 25e^{-0.2t}dt = \frac{25}{-0.2}\int e^w dw = -125e^w + C = -125e^{-0.2t} + C.$$

Check: $\frac{d}{dt}(-125e^{-0.2t} + C) = -125e^{-0.2t}(-0.2) = 25e^{-0.2t}$.

7. We use the substitution $w = t^2$, $dw = 2t\,dt$.

$$\int t\cos(t^2)dt = \frac{1}{2}\int \cos(w)dw = \frac{1}{2}\sin(w) + C = \frac{1}{2}\sin(t^2) + C.$$

Check: $\frac{d}{dt}(\frac{1}{2}\sin(t^2) + C) = \frac{1}{2}\cos(t^2)(2t) = t\cos(t^2)$.

8. We use the substitution $w = 2x$, $dw = 2\,dx$.

$$\int \sin(2x)dx = \frac{1}{2}\int \sin(w)dw = -\frac{1}{2}\cos(w) + C = -\frac{1}{2}\cos(2x) + C.$$

Check: $\frac{d}{dx}(-\frac{1}{2}\cos(2x) + C) = \frac{1}{2}\sin(2x)(2) = \sin(2x)$.

9. We use the substitution $w = 3 - t$, $dw = -\,dt$.

$$\int \sin(3-t)dt = -\int \sin(w)\,dw = -(-\cos(w)) + C = \cos(3-t) + C.$$

Check: $\frac{d}{dt}(\cos(3-t) + C) = -\sin(3-t)(-1) = \sin(3-t)$.

10. We use the substitution $w = -x^2$, $dw = -2x\,dx$.

$$\begin{aligned}\int xe^{-x^2}\,dx &= -\frac{1}{2}\int e^{-x^2}(-2x\,dx) = -\frac{1}{2}\int e^w\,dw \\ &= -\frac{1}{2}e^w + C = -\frac{1}{2}e^{-x^2} + C.\end{aligned}$$

Check: $\frac{d}{dx}(-\frac{1}{2}e^{-x^2} + C) = (-2x)(-\frac{1}{2}e^{-x^2}) = xe^{-x^2}$.

11. Either expand $(r+1)^3$ or use the substitution $w = r+1$. If $w = r+1$, then $dw = dr$ and

$$\int (r+1)^3\,dr = \int w^3\,dw = \frac{1}{4}w^4 + C = \frac{1}{4}(r+1)^4 + C.$$

12. We use the substitution $w = y^2 + 5$, $dw = 2y\,dy$.

$$\begin{aligned}\int y(y^2+5)^8\,dy &= \frac{1}{2}\int (y^2+5)^8(2y\,dy) \\ &= \frac{1}{2}\int w^8\,dw = \frac{1}{2}\frac{w^9}{9} + C \\ &= \frac{1}{18}(y^2+5)^9 + C.\end{aligned}$$

Check: $\frac{d}{dy}(\frac{1}{18}(y^2+5)^9 + C) = \frac{1}{18}[9(y^2+5)^8(2y)] = y(y^2+5)^8$.

13. We use the substitution $w = t^3 - 3$, $dw = 3t^2\,dt$.

$$\begin{aligned}\int t^2(t^3-3)^{10}\,dt &= \frac{1}{3}\int (t^3-3)^{10}(3t^2dt) = \int w^{10}\left(\frac{1}{3}\,dw\right) \\ &= \frac{1}{3}\frac{w^{11}}{11} + C = \frac{1}{33}(t^3-3)^{11} + C.\end{aligned}$$

Check: $\frac{d}{dt}[\frac{1}{33}(t^3-3)^{11} + C] = \frac{1}{3}(t^3-3)^{10}(3t^2) = t^2(t^3-3)^{10}$.

14. We use the substitution $w = 1 + 2x^3$, $dw = 6x^2\,dx$.

$$\int x^2(1+2x^3)^2\,dx = \int w^2(\frac{1}{6}\,dw) = \frac{1}{6}(\frac{w^3}{3}) + C = \frac{1}{18}(1+2x^3)^3 + C.$$

Check: $\frac{d}{dx}\left[\frac{1}{18}(1+2x^2)^3 + C\right] = \frac{1}{18}[3(1+2x^3)^2(6x^2)] = x^2(1+2x^3)^2$.

15. We use the substitution $w = x^2 + 3$, $dw = 2x\,dx$.

$$\int x(x^2+3)^2\,dx = \int w^2(\frac{1}{2}\,dw) = \frac{1}{2}\frac{w^3}{3} + C = \frac{1}{6}(x^2+3)^3 + C.$$

Check: $\frac{d}{dx}\left[\frac{1}{6}(x^2+3)^3 + C\right] = \frac{1}{6}\left[3(x^2+3)^2(2x)\right] = x(x^2+3)^2$.

16. We use the substitution $w = x^2 - 4$, $dw = 2x\,dx$.

$$\begin{aligned}\int x(x^2-4)^{7/2}\,dx &= \frac{1}{2}\int (x^2-4)^{7/2}(2xdx) = \frac{1}{2}\int w^{7/2}\,dw \\ &= \frac{1}{2}\left(\frac{2}{9}w^{9/2}\right) + C = \frac{1}{9}(x^2-4)^{9/2} + C.\end{aligned}$$

Check: $\frac{d}{dx}\left(\frac{1}{9}(x^2-4)^{9/2} + C\right) = \frac{1}{9}\left(\frac{9}{2}(x^2-4)^{7/2}\right)2x = x(x^2-4)^{7/2}$.

17. In this case, it seems easier not to substitute.

$$\int y^2(1+y)^2\,dy = \int y^2(y^2+2y+1)\,dy = \int (y^4+2y^3+y^2)\,dy$$
$$= \frac{y^5}{5}+\frac{y^4}{2}+\frac{y^3}{3}+C.$$

Check: $\frac{d}{dy}\left(\frac{y^5}{5}+\frac{y^4}{2}+\frac{y^3}{3}+C\right) = y^4+2y^3+y^2 = y^2(y+1)^2$.

18. We use the substitution $w = 2t-7$, $dw = 2\,dt$.

$$\int (2t-7)^{73}\,dt = \frac{1}{2}\int w^{73}\,dw = \frac{1}{(2)(74)}w^{74}+C = \frac{1}{148}(2t-7)^{74}+C.$$

Check: $\frac{d}{dt}\left[\frac{1}{148}(2t-7)^{74}+C\right] = \frac{74}{148}(2t-7)^{73}(2) = (2t-7)^{73}$.

19. We use the substitution $w = y+5$, $dw = dy$, to get

$$\int \frac{dy}{y+5} = \int \frac{dw}{w} = \ln|w|+C = \ln|y+5|+C.$$

Check: $\frac{d}{dy}(\ln|y+5|+C) = \frac{1}{y+5}$.

20. We use the substitution $w = 4-x$, $dw = -dx$.

$$\int \frac{1}{\sqrt{4-x}}\,dx = -\int \frac{1}{\sqrt{w}}\,dw = -2\sqrt{w}+C = -2\sqrt{4-x}+C.$$

Check: $\frac{d}{dx}(-2\sqrt{4-x}+C) = -2\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{4-x}}\cdot -1 = \frac{1}{\sqrt{4-x}}$.

21. In this case, it seems easier not to substitute.

$$\int (x^2+3)^2\,dx = \int (x^4+6x^2+9)\,dx = \frac{x^5}{5}+2x^3+9x+C.$$

Check: $\frac{d}{dx}\left[\frac{x^5}{5}+2x^3+9x+C\right] = x^4+6x^2+9 = (x^2+3)^2$.

22. We use the substitution $w = x^3+1$, $dw = 3x^2\,dx$, to get

$$\int x^2e^{x^3+1}\,dx = \frac{1}{3}\int e^w\,dw = \frac{1}{3}e^w+C = \frac{1}{3}e^{x^3+1}+C.$$

Check: $\frac{d}{dx}\left(\frac{1}{3}e^{x^3+1}+C\right) = \frac{1}{3}e^{x^3+1}\cdot 3x^2 = x^2e^{x^3+1}$.

23. We use the substitution $w = \cos\theta+5$, $dw = -\sin\theta\,d\theta$.

$$\int \sin\theta(\cos\theta+5)^7\,d\theta = -\int w^7\,dw = -\frac{1}{8}w^8+C$$
$$= -\frac{1}{8}(\cos\theta+5)^8+C.$$

Check:

$$\frac{d}{d\theta}\left[-\frac{1}{8}(\cos\theta+5)^8+C\right] = -\frac{1}{8}\cdot 8(\cos\theta+5)^7\cdot(-\sin\theta)$$
$$= \sin\theta(\cos\theta+5)^7$$

24. We use the substitution $w = \cos 3t,\ dw = -3\sin 3t\,dt$.

$$\int \sqrt{\cos 3t}\sin 3t\,dt = -\frac{1}{3}\int \sqrt{w}\,dw$$
$$= -\frac{1}{3}\cdot\frac{2}{3}w^{\frac{3}{2}} + C = -\frac{2}{9}(\cos 3t)^{\frac{3}{2}} + C.$$

Check:

$$\frac{d}{dt}\left[-\frac{2}{9}(\cos 3t)^{\frac{3}{2}} + C\right] = -\frac{2}{9}\cdot\frac{3}{2}(\cos 3t)^{\frac{1}{2}}\cdot(-\sin 3t)\cdot 3$$
$$= \sqrt{\cos 3t}\sin 3t.$$

25. We use the substitution $w = \sin\theta,\ dw = \cos\theta\,d\theta$.

$$\int \sin^6\theta\cos\theta\,d\theta = \int w^6\,dw = \frac{w^7}{7} + C = \frac{\sin^7\theta}{7} + C.$$

Check: $\dfrac{d}{d\theta}\left[\dfrac{\sin^7\theta}{7} + C\right] = \sin^6\theta\cos\theta.$

26. We use the substitution $w = \sin\alpha,\ dw = \cos\alpha\,d\alpha$.

$$\int \sin^3\alpha\cos\alpha\,d\alpha = \int w^3\,dw = \frac{w^4}{4} + C = \frac{\sin^4\alpha}{4} + C.$$

Check: $\dfrac{d}{d\alpha}\left(\dfrac{\sin^4\alpha}{4} + C\right) = \dfrac{1}{4}\cdot 4\sin^3\alpha\cdot\cos\alpha = \sin^3\alpha\cos\alpha.$

27. We use the substitution $w = \sin 5\theta,\ dw = 5\cos 5\theta\,d\theta$.

$$\int \sin^6 5\theta\cos 5\theta\,d\theta = \frac{1}{5}\int w^6\,dw = \frac{1}{5}(\frac{w^7}{7}) + C = \frac{1}{35}\sin^7 5\theta + C.$$

Check: $\dfrac{d}{d\theta}(\dfrac{1}{35}\sin^7 5\theta + C) = \dfrac{1}{35}[7\sin^6 5\theta](5\cos 5\theta) = \sin^6 5\theta\cos 5\theta.$
Note that we could also use Problem 25 to solve this problem, substituting $w = 5\theta$ and $dw = 5\,d\theta$ to get:

$$\int \sin^6 5\theta\cos 5\theta\,d\theta = \frac{1}{5}\int \sin^6 w\cos w\,dw$$
$$= \frac{1}{5}(\frac{\sin^7 w}{7}) + C = \frac{1}{35}\sin^7 5\theta + C.$$

28. We use the substitution $w = \cos 2x,\ dw = -2\sin 2x\,dx$.

$$\int \tan 2x\,dx = \int \frac{\sin 2x}{\cos 2x}\,dx = -\frac{1}{2}\int \frac{dw}{w}$$
$$= -\frac{1}{2}\ln|w| + C = -\frac{1}{2}\ln|\cos 2x| + C.$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{2}\ln|\cos 2x| + C\right] = -\frac{1}{2}\cdot\frac{1}{\cos 2x}\cdot -2\sin 2x$$
$$= \frac{\sin 2x}{\cos 2x} = \tan 2x.$$

29. We use the substitution $w = \ln z, dw = \frac{1}{z}\,dz$.

$$\int \frac{(\ln z)^2}{z}\,dz = \int w^2\,dw = \frac{w^3}{3} + C = \frac{(\ln z)^3}{3} + C.$$

Check: $\frac{d}{dz}\left[\frac{(\ln z)^3}{3} + C\right] = 3 \cdot \frac{1}{3}(\ln z)^2 \cdot \frac{1}{z} = \frac{(\ln z)^2}{z}$.

30. We use the substitution $w = e^t + t,\ dw = (e^t + 1)\,dt$.

$$\int \frac{e^t + 1}{e^t + t}\,dt = \int \frac{1}{w}\,dw = \ln|w| + C = \ln|e^t + t| + C.$$

Check: $\frac{d}{dt}(\ln|e^t + t| + C) = \frac{e^t + 1}{e^t + t}$.

31. We use the substitution $w = y^2 + 4,\ dw = 2y\,dy$.

$$\int \frac{y}{y^2 + 4}\,dy = \frac{1}{2}\int \frac{dw}{w} = \frac{1}{2}\ln|w| + C = \frac{1}{2}\ln(y^2 + 4) + C.$$

(We can drop the absolute value signs since $y^2 + 4 \geq 0$ for all y.)

Check: $\frac{d}{dy}\left[\frac{1}{2}\ln(y^2 + 4) + C\right] = \frac{1}{2} \cdot \frac{1}{y^2 + 4} \cdot 2y = \frac{y}{y^2 + 4}$.

32. We use the substitution $w = \sqrt{x},\ dw = \frac{1}{2\sqrt{x}}\,dx$.

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = \int \cos w(2\,dw) = 2\sin w + C = 2\sin\sqrt{x} + C.$$

Check: $\frac{d}{dx}(2\sin\sqrt{x} + C) = 2\cos\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right) = \frac{\cos\sqrt{x}}{\sqrt{x}}$.

33. We use the substitution $w = \sqrt{y},\ dw = \frac{1}{2\sqrt{y}}\,dy$.

$$\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy = 2\int e^w\,dw = 2e^w + C = 2e^{\sqrt{y}} + C.$$

Check: $\frac{d}{dy}(2e^{\sqrt{y}} + C) = 2e^{\sqrt{y}} \cdot \frac{1}{2\sqrt{y}} = \frac{e^{\sqrt{y}}}{\sqrt{y}}$.

34. We use the substitution $w = x + e^x,\ dw = (1 + e^x)\,dx$.

$$\int \frac{1 + e^x}{\sqrt{x + e^x}}\,dx = \int \frac{dw}{\sqrt{w}} = 2\sqrt{w} + C = 2\sqrt{x + e^x} + C.$$

Check: $\frac{d}{dx}(2\sqrt{x + e^x} + C) = 2 \cdot \frac{1}{2}(x + e^x)^{-\frac{1}{2}} \cdot (1 + e^x) = \frac{1 + e^x}{\sqrt{x + e^x}}$.

35. We use the substitution $w = 2 + e^x,\ dw = e^x\,dx$.

$$\int \frac{e^x}{2 + e^x}\,dx = \int \frac{dw}{w} = \ln|w| + C = \ln(2 + e^x) + C.$$

(We can drop the absolute value signs since $2 + e^x \geq 0$ for all x.)

Check: $\frac{d}{dx}[\ln(2 + e^x) + C] = \frac{1}{2 + e^x} \cdot e^x = \frac{e^x}{2 + e^x}$.

36. We use the substitution $w = x^2 + 2x + 19,\ dw = 2(x + 1)dx$.

$$\int \frac{(x + 1)\,dx}{x^2 + 2x + 19} = \frac{1}{2}\int \frac{dw}{w} = \frac{1}{2}\ln|w| + C = \frac{1}{2}\ln(x^2 + 2x + 19) + C.$$

(We can drop the absolute value signs, since $x^2 + 2x + 19 = (x + 1)^2 + 18 > 0$ for all x.)

Check: $\frac{1}{dx}[\frac{1}{2}\ln(x^2 + 2x + 19)] = \frac{1}{2}\frac{1}{x^2 + 2x + 19}(2x + 2) = \frac{x + 1}{x^2 + 2x + 19}$.

109. (a) At time $t = 0$, the rate of oil leakage $= r(0) = 50$ thousand liters/minute.
At $t = 60$, rate $= r(60) = 15.06$ thousand liters/minute.

(b) To find the amount of oil leaked during the first hour, we integrate the rate from $t = 0$ to $t = 60$:

$$\begin{aligned} \text{Oil leaked} &= \int_0^{60} 50e^{-0.02t}\,dt = \left(-\frac{50}{0.02}e^{-0.02t}\right)\Bigg|_0^{60} \\ &= -2500e^{-1.2} + 2500e^{0} = 1747 \text{ thousand liters.} \end{aligned}$$