Calculus 112 Practice Problems

Section 7.2 Problems #3-30, #39

3. Let $u = t$, $v' = \sin t$. Thus, $v = -\cos t$ and $u' = 1$. With this choice of u and v, integration by parts gives:

$$\begin{aligned}\int t \sin t\, dt &= -t \cos t - \int (-\cos t)\, dt \\ &= -t\cos t + \sin t + C.\end{aligned}$$

4. Let $u = t^2$, $v' = \sin t$ implying $v = -\cos t$ and $u' = 2t$. Integrating by parts, we get:

$$\int t^2 \sin t\, dt = -t^2 \cos t - \int 2t(-\cos t)\, dt.$$

Again, applying integration by parts with $u = t$, $v' = \cos t$, we have:

$$\int t \cos t\, dt = t \sin t + \cos t + C.$$

Thus

$$\int t^2 \sin t\, dt = -t^2 \cos t + 2t \sin t + 2\cos t + C.$$

5. Let $u = t$ and $v' = e^{5t}$, so $u' = 1$ and $v = \frac{1}{5}e^{5t}$.
Then $\int te^{5t}\, dt = \frac{1}{5}te^{5t} - \int \frac{1}{5}e^{5t}\, dt = \frac{1}{5}te^{5t} - \frac{1}{25}e^{5t} + C.$

6. Let $u = t^2$ and $v' = e^{5t}$, so $u' = 2t$ and $v = \frac{1}{5}e^{5t}$.

Then $\int t^2 e^{5t}\,dt = \frac{1}{5}t^2e^{5t} - \frac{2}{5}\int te^{5t}\,dt$.

Using Problem 5, we have $\int t^2e^{5t}\,dt = \frac{1}{5}t^2e^{5t} - \frac{2}{5}(\frac{1}{5}te^{5t} - \frac{1}{25}e^{5t}) + C$
$= \frac{1}{5}t^2e^{5t} - \frac{2}{25}te^{5t} + \frac{2}{125}e^{5t} + C$.

7. Let $u = p$ and $v' = e^{(-0.1)p}$, $u' = 1$. Thus, $v = \int e^{(-0.1)p}\,dp = -10e^{(-0.1)p}$. With this choice of u and v, integration by parts gives:

$$\begin{aligned}\int pe^{(-0.1)p}\,dp &= p(-10e^{(-0.1)p}) - \int(-10e^{(-0.1)p})\,dp\\ &= -10pe^{(-0.1)p} + 10\int e^{(-0.1)p}\,dp\\ &= -10pe^{(-0.1)p} - 100e^{(-0.1)p} + C.\end{aligned}$$

8. Let $u = z+1$, $v' = e^{2z}$. Thus, $v = \frac{1}{2}e^{2z}$ and $u' = 1$. Integrating by parts, we get:

$$\begin{aligned}\int(z+1)e^{2z}\,dz &= (z+1)\cdot\frac{1}{2}e^{2z} - \int\frac{1}{2}e^{2z}\,dz\\ &= \frac{1}{2}(z+1)e^{2z} - \frac{1}{4}e^{2z} + C\\ &= \frac{1}{4}(2z+1)e^{2z} + C.\end{aligned}$$

9. Let $u = \ln y$, $v' = y$. Then, $v = \frac{1}{2}y^2$ and $u' = \dfrac{1}{y}$. Integrating by parts, we get:

$$\begin{aligned}\int y\ln y\,dy &= \frac{1}{2}y^2\ln y - \int\frac{1}{2}y^2\cdot\frac{1}{y}\,dy\\ &= \frac{1}{2}y^2\ln y - \frac{1}{2}\int y\,dy\\ &= \frac{1}{2}y^2\ln y - \frac{1}{4}y^2 + C.\end{aligned}$$

10. Let $u = \ln x$ and $v' = x^3$, so $u' = \frac{1}{x}$ and $v = \frac{x^4}{4}$. Then

$$\int x^3\ln x\,dx = \frac{x^4}{4}\ln x - \int\frac{x^3}{4}\,dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C.$$

11. Let $u = \ln 5q$, $v' = q^5$. Then $v = \frac{1}{6}q^6$ and $u' = \dfrac{1}{q}$. Integrating by parts, we get:

$$\begin{aligned}\int q^5\ln 5q\,dq &= \frac{1}{6}q^6\ln 5q - \int(5\cdot\frac{1}{5q})\cdot\frac{1}{6}q^6\,dq\\ &= \frac{1}{6}q^6\ln 5q - \frac{1}{36}q^6 + C.\end{aligned}$$

12. Let $u = \theta^2$ and $v' = \cos 3\theta$, so $u' = 2\theta$ and $v = \frac{1}{3}\sin 3\theta$.

Then $\int\theta^2\cos 3\theta\,d\theta = \frac{1}{3}\theta^2\sin 3\theta - \frac{2}{3}\int\theta\sin 3\theta\,d\theta$. The integral on the right hand side is simpler than our original integral, but to evaluate it we need to again use integration by parts.

To find $\int\theta\sin 3\theta\,d\theta$, let $u = \theta$ and $v' = \sin 3\theta$, so $u' = 1$ and $v = -\frac{1}{3}\cos 3\theta$.

This gives

$$\int\theta\sin 3\theta\,d\theta = -\frac{1}{3}\theta\cos 3\theta + \frac{1}{3}\int\cos 3\theta\,d\theta = -\frac{1}{3}\theta\cos 3\theta + \frac{1}{9}\sin 3\theta + C.$$

Thus,

$$\int\theta^2\cos 3\theta d\theta = \frac{1}{3}\theta^2\sin 3\theta + \frac{2}{9}\theta\cos 3\theta - \frac{2}{27}\sin 3\theta + C.$$

13. Let $u = \sin\theta$ and $v' = \sin\theta$, so $u' = \cos\theta$ and $v = -\cos\theta$. Then

$$\begin{aligned}\int \sin^2\theta\, d\theta &= -\sin\theta\cos\theta + \int \cos^2\theta\, d\theta \\ &= -\sin\theta\cos\theta + \int (1-\sin^2\theta)\, d\theta \\ &= -\sin\theta\cos\theta + \int 1\, d\theta - \int \sin^2\theta\, d\theta.\end{aligned}$$

By adding $\int \sin^2\theta\, d\theta$ to both sides of the above equation, we find that $2\int \sin^2\theta\, d\theta = -\sin\theta\cos\theta + \theta + C$, so $\int \sin^2\theta\, d\theta = -\frac{1}{2}\sin\theta\cos\theta + \frac{\theta}{2} + C'$.

14. Let $u = \cos(3\alpha+1)$ and $v' = \cos(3\alpha+1)$, so $u' = -3\sin(3\alpha+1)$, and $v = \frac{1}{3}\sin(3\alpha+1)$. Then

$$\begin{aligned}\int \cos^2(3\alpha+1)\, d\alpha &= \int (\cos(3\alpha+1))\cos(3\alpha+1)\, d\alpha \\ &= \frac{1}{3}\cos(3\alpha+1)\sin(3\alpha+1) + \int \sin^2(3\alpha+1)\, d\alpha \\ &= \frac{1}{3}\cos(3\alpha+1)\sin(3\alpha+1) + \int \left(1-\cos^2(3\alpha+1)\right)\, d\alpha \\ &= \frac{1}{3}\cos(3\alpha+1)\sin(3\alpha+1) + \alpha - \int \cos^2(3\alpha+1)\, d\alpha.\end{aligned}$$

By adding $\int \cos^2(3\alpha+1)\, d\alpha$ to both sides of the above equation, we find that

$$2\int \cos^2(3\alpha+1)\, d\alpha = \frac{1}{3}\cos(3\alpha+1)\sin(3\alpha+1) + \alpha + C,$$

which gives

$$\int \cos^2(3\alpha+1)\, d\alpha = \frac{1}{6}\cos(3\alpha+1)\sin(3\alpha+1) + \frac{\alpha}{2} + C.$$

15. Let $u = (\ln t)^2$ and $v' = 1$, so $u' = \dfrac{2\ln t}{t}$ and $v = t$. Then

$$\int (\ln t)^2\, dt = t(\ln t)^2 - 2\int \ln t\, dt = t(\ln t)^2 - 2t\ln t + 2t + C.$$

(We use the fact that $\displaystyle\int \ln x\, dx = x\ln x - x + C$, a result which can be derived using integration by parts.)

16. Let $u = y$ and $v' = (y+3)^{1/2}$, so $u' = 1$ and $v = \frac{2}{3}(y+3)^{3/2}$:

$$\int y\sqrt{y+3}\, dy = \frac{2}{3}y(y+3)^{3/2} - \int \frac{2}{3}(y+3)^{3/2}\, dy = \frac{2}{3}y(y+3)^{3/2} - \frac{4}{15}(y+3)^{5/2} + C.$$

17. Let $u = t+2$ and $v' = \sqrt{2+3t}$, so $u' = 1$ and $v = \frac{2}{9}(2+3t)^{3/2}$. Then

$$\begin{aligned}\int (t+2)\sqrt{2+3t}\, dt &= \frac{2}{9}(t+2)(2+3t)^{3/2} - \frac{2}{9}\int (2+3t)^{3/2}\, dt \\ &= \frac{2}{9}(t+2)(2+3t)^{3/2} - \frac{4}{135}(2+3t)^{5/2} + C.\end{aligned}$$

18. Let $u = \theta+1$ and $v' = \sin(\theta+1)$, so $u' = 1$ and $v = -\cos(\theta+1)$.

$$\begin{aligned}\int (\theta+1)\sin(\theta+1)\, d\theta &= -(\theta+1)\cos(\theta+1) + \int \cos(\theta+1)\, d\theta \\ &= -(\theta+1)\cos(\theta+1) + \sin(\theta+1) + C.\end{aligned}$$

19. Let $u = z$, $v' = e^{-z}$. Thus $v = -e^{-z}$ and $u' = 1$. Integration by parts gives:

$$\begin{aligned}\int ze^{-z}\,dz &= -ze^{-z} - \int(-e^{-z})\,dz\\ &= -ze^{-z} - e^{-z} + C\\ &= -(z+1)e^{-z} + C.\end{aligned}$$

20. Let $u = \ln x$, $v' = x^{-2}$. Then $v = -x^{-1}$ and $u' = x^{-1}$. Integrating by parts, we get:

$$\begin{aligned}\int x^{-2}\ln x\,dx &= -x^{-1}\ln x - \int(-x^{-1})\cdot x^{-1}\,dx\\ &= -x^{-1}\ln x - x^{-1} + C.\end{aligned}$$

21. Let $u = y$ and $v' = \frac{1}{\sqrt{5-y}}$, so $u' = 1$ and $v = -2(5-y)^{1/2}$.

$$\int \frac{y}{\sqrt{5-y}}\,dy = -2y(5-y)^{1/2} + 2\int(5-y)^{1/2}\,dy = -2y(5-y)^{1/2} - \frac{4}{3}(5-y)^{3/2} + C.$$

22. $\displaystyle\int \frac{t+7}{\sqrt{5-t}}\,dt = \int \frac{t}{\sqrt{5-t}}\,dt + 7\int(5-t)^{-1/2}\,dt.$

To calculate the first integral, we use integration by parts. Let $u = t$ and $v' = \frac{1}{\sqrt{5-t}}$, so $u' = 1$ and $v = -2(5-t)^{1/2}$. Then

$$\int \frac{t}{\sqrt{5-t}}\,dt = -2t(5-t)^{1/2} + 2\int(5-t)^{1/2}\,dt = -2t(5-t)^{1/2} - \frac{4}{3}(5-t)^{3/2} + C.$$

We can calculate the second integral directly: $7\displaystyle\int(5-t)^{-1/2} = -14(5-t)^{1/2} + C_1$. Thus

$$\int \frac{t+7}{\sqrt{5-t}}\,dt = -2t(5-t)^{1/2} - \frac{4}{3}(5-t)^{3/2} - 14(5-t)^{1/2} + C_2.$$

23. Let $u = (\ln x)^4$ and $v' = x$, so $u' = \frac{4(\ln x)^3}{x}$ and $v = \frac{x^2}{2}$. Then

$$\int x(\ln x)^4\,dx = \frac{x^2(\ln x)^4}{2} - 2\int x(\ln x)^3\,dx.$$

$\int x(\ln x)^3\,dx$ is somewhat less complicated than $\int x(\ln x)^4\,dx$. To calculate it, we again try integration by parts, this time letting $u = (\ln x)^3$ (instead of $(\ln x)^4$) and $v' = x$. We find

$$\int x(\ln x)^3\,dx = \frac{x^2}{2}(\ln x)^3 - \frac{3}{2}\int x(\ln x)^2\,dx.$$

Once again, express the given integral in terms of a less-complicated one. Using integration by parts two more times, we find that

$$\int x(\ln x)^2\,dx = \frac{x^2}{2}(\ln x)^2 - \int x(\ln x)\,dx$$

and that

$$\int x\ln x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C.$$

Putting this all together, we have

$$\int x(\ln x)^4\,dx = \frac{x^2}{2}(\ln x)^4 - x^2(\ln x)^3 + \frac{3}{2}x^2(\ln x)^2 - \frac{3}{2}x^2\ln x + \frac{3}{4}x^2 + C.$$

24. Let $u = \arcsin w$ and $v' = 1$, so $u' = \frac{1}{\sqrt{1-w^2}}$ and $v = w$. Then

$$\int \arcsin w \, dw = w \arcsin w - \int \frac{w}{\sqrt{1-w^2}} \, dw = w \arcsin w + \sqrt{1-w^2} + C.$$

25. Let $u = \arctan 7z$ and $v' = 1$, so $u' = \frac{7}{1+49z^2}$ and $v = z$. Now $\int \frac{7z\,dz}{1+49z^2}$ can be evaluated by the substitution $w = 1 + 49z^2$, $dw = 98z\,dz$, so

$$\int \frac{7z\,dz}{1+49z^2} = 7 \int \frac{\frac{1}{98}\,dw}{w} = \frac{1}{14} \int \frac{dw}{w} = \frac{1}{14} \ln |w| + C = \frac{1}{14} \ln(1+49z^2) + C$$

So

$$\int \arctan 7z \, dz = z \arctan 7z - \frac{1}{14} \ln(1+49z^2) + C.$$

26. This integral can first be simplified by making the substitution $w = x^2$, $dw = 2x\,dx$. Then

$$\int x \arctan x^2 \, dx = \frac{1}{2} \int \arctan w \, dw.$$

To evaluate $\int \arctan w \, dw$, we'll use integration by parts. Let $u = \arctan w$ and $v' = 1$, so $u' = \frac{1}{1+w^2}$ and $v = w$. Then

$$\int \arctan w \, dw = w \arctan w - \int \frac{w}{1+w^2} \, dw = w \arctan w - \frac{1}{2} \ln |1 + w^2| + C.$$

Since $1 + w^2$ is never negative, we can drop the absolute value signs. Thus, we have

$$\begin{aligned} \int x \arctan x^2 \, dx &= \frac{1}{2} \left(x^2 \arctan x^2 - \frac{1}{2} \ln(1 + (x^2)^2) + C \right) \\ &= \frac{1}{2} x^2 \arctan x^2 - \frac{1}{4} \ln(1 + x^4) + C. \end{aligned}$$

27. Let $u = x^2$ and $v' = xe^{x^2}$, so $u' = 2x$ and $v = \frac{1}{2}e^{x^2}$. Then

$$\int x^3 e^{x^2} \, dx = \frac{1}{2} x^2 e^{x^2} - \int x e^{x^2} \, dx = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C.$$

Note that we can also do this problem by substitution and integration by parts. If we let $w = x^2$, so $dw = 2x\,dx$, then $\displaystyle\int x^3 e^{x^2} \, dx = \frac{1}{2} \int w e^w \, dw$. We could then perform integration by parts on this integral to get the same result.

28. To simplify matters, let us try the substitution $w = x^3$, $dw = 3x^2\,dx$. Then

$$\int x^5 \cos x^3 \, dx = \frac{1}{3} \int w \cos w \, dw.$$

Now we integrate by parts. Let $u = w$ and $v' = \cos w$, so $u' = 1$ and $v = \sin w$. Then

$$\begin{aligned} \frac{1}{3} \int w \cos w \, dw &= \frac{1}{3} [w \sin w - \int \sin w \, dw] \\ &= \frac{1}{3} [w \sin w + \cos w] + C \\ &= \frac{1}{3} x^3 \sin x^3 + \frac{1}{3} \cos x^3 + C \end{aligned}$$

29. Let $u = x$, $u' = 1$ and $v' = \sinh x$, $v = \cosh x$. Integrating by parts, we get

$$\begin{aligned} \int x \sinh x \, dx &= x \cosh x - \int \cosh x \, dx \\ &= x \cosh x - \sinh x + C. \end{aligned}$$

30. Let $u = x - 1, u' = 1$ and $v' = \cosh x, v = \sinh x$. Integrating by parts, we get

$$\begin{aligned}\int (x-1)\cosh x\,dx &= (x-1)\sinh x - \int \sinh x\,dx \\ &= (x-1)\sinh x - \cosh x + C.\end{aligned}$$

39. (a) This integral can be evaluated using integration by parts with $u = x$, $v' = \sin x$.
(b) We evaluate this integral using the substitution $w = 1 + x^3$.
(c) We evaluate this integral using the substitution $w = x^2$.
(d) We evaluate this integral using the substitution $w = x^3$.
(e) We evaluate this integral using the substitution $w = 3x + 1$.
(f) This integral can be evaluated using integration by parts with $u = x^2$, $v' = \sin x$.
(g) This integral can be evaluated using integration by parts with $u = \ln x$, $v' = 1$.