Calculus 112 Practice Problems

Section 7.5 Problems #11, #14, #19

11. (a) (i) Let $f(x) = \frac{1}{1+x^2}$. The left-hand Riemann sum is

$$\begin{aligned}
&\frac{1}{8}\left(f(0) + f\left(\frac{1}{8}\right) + f\left(\frac{2}{8}\right) + \cdots + f\left(\frac{7}{8}\right)\right) \\
&= \frac{1}{8}\left(\frac{64}{64} + \frac{64}{65} + \frac{64}{68} + \frac{64}{73} + \frac{64}{80} + \frac{64}{89} + \frac{64}{100} + \frac{64}{113}\right) \\
&\approx 8(0.1020) = 0.8160.
\end{aligned}$$

(ii) Let $f(x) = \frac{1}{1+x^2}$. The right-hand Riemann sum is

$$\begin{aligned}
&\frac{1}{8}\left(f\left(\frac{1}{8}\right) + f\left(\frac{2}{8}\right) + f\left(\frac{3}{8}\right) + \cdots + f(1)\right) \\
&= \frac{1}{8}\left(\frac{64}{65} + \frac{64}{68} + \frac{64}{73} + \frac{64}{80} + \frac{64}{89} + \frac{64}{100} + \frac{64}{113} + \frac{64}{128}\right) \\
&\approx 0.8160 - \frac{1}{16} = 0.7535.
\end{aligned}$$

(iii) The trapezoid rule gives us that

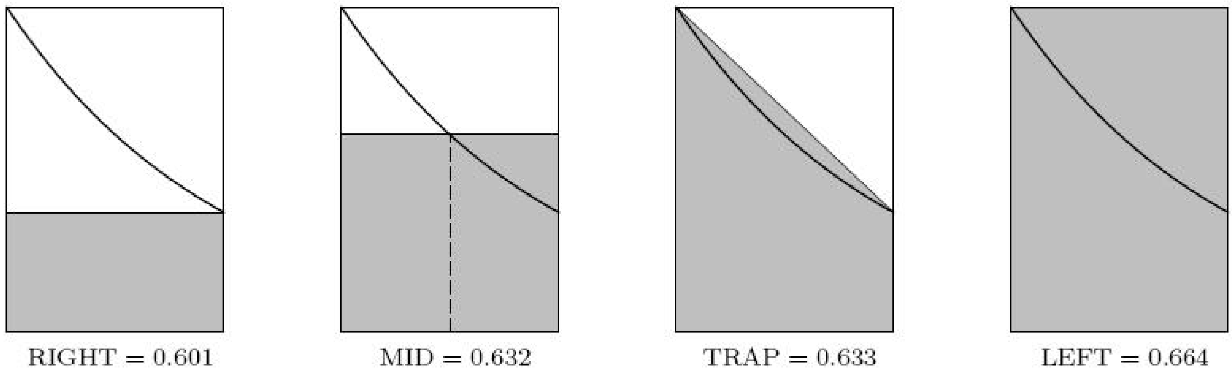
$$\text{TRAP}(8) = \frac{\text{LEFT}(8) + \text{RIGHT}(8)}{2} \approx 0.7847.$$

(b) Since $1 + x^2$ is increasing for $x > 0$, so $\dfrac{1}{1+x^2}$ is decreasing over the interval. Thus

$$\begin{aligned}
\text{RIGHT}(8) < \int_0^1 \frac{1}{1+x^2}\,dx < \text{LEFT}(8) \\
0.7535 < \frac{\pi}{4} < 0.8160 \\
3.014 < \pi < 3.264.
\end{aligned}$$

14. For a decreasing function whose graph is concave up, the diagrams below show that RIGHT $<$ MID $<$ TRAP $<$ LEFT. Thus,

(a) $0.664 =$ LEFT, $0.633 =$ TRAP, $0.632 =$ MID, and $0.601 =$ RIGHT.
(b) $0.632 <$ true value < 0.633.



19. (a) Since $f(x)$ is closer to horizontal (that is, $|f'| < |g'|$), LEFT and RIGHT will be more accurate with $f(x)$.
(b) Since $g(x)$ has more curvature, MID and TRAP will be more accurate with $f(x)$.