Calculus 112 Practice Problems

Section 7.7 Problems #5, #6, #7, #9, #13, #15

5. We have

$$\int_1^\infty \frac{1}{5x+2}dx = \lim_{b\to\infty}\int_1^b \frac{1}{5x+2}dx = \lim_{b\to\infty}\left(\frac{1}{5}\ln(5x+2)\right)\Big|_1^b = \lim_{b\to\infty}\left(\frac{1}{5}\ln(5b+2) - \frac{1}{5}\ln(7)\right).$$

As $b \leftarrow \infty$, we know that $\ln(5b+2) \to \infty$, and so this integral diverges.

6. We have

$$\int_1^\infty \frac{1}{(x+2)^2}dx = \lim_{b\to\infty}\int_1^b \frac{1}{(x+2)^2}dx = \lim_{b\to\infty}\left(\frac{-1}{x+2}\right)\Big|_1^b = \lim_{b\to\infty}\left(\frac{-1}{b+2} - \frac{-1}{3}\right) = 0 + \frac{1}{3} = \frac{1}{3}.$$

This integral converges to $1/3$.

7. This integral is improper at the lower end, so

$$\begin{aligned}
\int_0^1 \ln x\,dx &= \lim_{a\to0^+}\int_a^1 \ln x\,dx \\
&= \lim_{a\to0^+}(x\ln x - x)\Big|_a^1 \\
&= \lim_{a\to0^+}(1\ln 1 - 1) - (a\ln a - a)) \\
&= -1 + \lim_{a\to0^+} a(1-\ln a) \\
&= -1 + \lim_{a\to0^+}\frac{1-\ln a}{a^{-1}} \\
&= -1 + \lim_{a\to0^+}\frac{-1/a}{-1/a^2} \quad \text{by l'Hopital} \\
&= -1 + \lim_{a\to0^+} a \\
&= -1.
\end{aligned}$$

If the integral converges, we'd expect it to have a negative value because the logarithm graph is below the x-axis for $0 < x < 1$.

9. We have

$$\int_0^\infty xe^{-x^2}dx = \lim_{b\to\infty}\int_0^b xe^{-x^2}dx = \lim_{b\to\infty}\left(\frac{-1}{2}e^{-x^2}\right)\Big|_0^b = \lim_{b\to\infty}\left(\frac{-1}{2}e^{-b^2} - \frac{-1}{2}\right) = 0 + \frac{1}{2} = \frac{1}{2}.$$

This integral converges to $1/2$.

13.

$$\begin{aligned}\int_{-\infty}^{0} \frac{e^x}{1+e^x}\,dx &= \lim_{b\to-\infty} \int_{b}^{0} \frac{e^x}{1+e^x}\,dx \\ &= \lim_{b\to-\infty} \ln|1+e^x|\Big|_b^0 \\ &= \lim_{b\to-\infty} [\ln|1+e^0| - \ln|1+e^b|] \\ &= \ln(1+1) - \ln(1+0) = \ln 2.\end{aligned}$$

15. This is an improper integral because $\sqrt{16-x^2} = 0$ at $x = 4$. So

$$\begin{aligned}\int_0^4 \frac{dx}{\sqrt{16-x^2}} &= \lim_{b\to 4^-} \int_0^b \frac{dx}{\sqrt{16-x^2}} \\ &= \lim_{b\to 4^-} (\arcsin x/4)\Big|_0^b \\ &= \lim_{b\to 4^-} [\arcsin(b/4) - \arcsin(0)] = \pi/2 - 0 = \pi/2.\end{aligned}$$