Calculus 112 Practice Problems

Section 7.8 Problems #11, #16, #19

11. Since $\dfrac{1}{1+x} \geq \dfrac{1}{2x}$ and $\dfrac{1}{2}\displaystyle\int_0^\infty \frac{1}{x}\,dx$ diverges, we have that $\displaystyle\int_1^\infty \frac{dx}{1+x}$ diverges.

16. Since we know the antiderivative of $\dfrac{1}{1+u^2}$, we can use the Fundamental Theorem of Calculus to evaluate the integral. Since the integrand is even, we write

$$\int_{-\infty}^{\infty} \frac{du}{1+u^2} = 2\int_0^\infty \frac{du}{1+u^2} = 2\lim_{b\to\infty}\int_0^b \frac{du}{1+u^2}$$
$$= 2\lim_{b\to\infty} \arctan b = 2\left(\frac{\pi}{2}\right) = \pi.$$

Thus, the integral converges to π.

19. For $\theta \geq 2$, we have $\dfrac{1}{\sqrt{\theta^3+1}} \leq \dfrac{1}{\sqrt{\theta^3}} = \dfrac{1}{\theta^{\frac{3}{2}}}$, and $\displaystyle\int_2^\infty \frac{d\theta}{\theta^{3/2}}$ converges (check by integration), so $\displaystyle\int_2^\infty \frac{d\theta}{\sqrt{\theta^3+1}}$ converges.