Calculus 112 Practice Problems

Section 8.1 Problems #9, #10, #11, #21

9. Each slice is a circular disk with radius $r = 2$ cm.

$$\text{Volume of disk } = \pi r^2 \Delta x = 4\pi \Delta x \text{ cm}^3.$$

Summing over all disks, we have

$$\text{Total volume } \approx \sum 4\pi \Delta x \text{ cm}^3.$$

Taking a limit as $\Delta x \to 0$, we get

$$\text{Total volume } = \lim_{\Delta x \to 0} \sum 4\pi \Delta x = \int_0^9 4\pi \, dx \text{ cm}^3.$$

Evaluating gives

$$\text{Total volume } = 4\pi x \Big|_0^9 = 36\pi \text{ cm}^3.$$

Check: The volume of the cylinder can also be calculated using the formula $V = \pi r^2 h = \pi 2^2 \cdot 9 = 36\pi \text{ cm}^3$.

10. Each slice is a circular disk. Since the radius of the cone is 2 cm and the length is 6 cm, the radius is one-third of the distance from the vertex. Thus, the radius at x is $r = x/3$ cm. See Figure 8.1.

$$\text{Volume of slice } \approx \pi r^2 \Delta x = \frac{\pi x^2}{9} \Delta x \text{ cm}^3.$$

Summing over all disks, we have

$$\text{Total volume } \approx \sum \pi \frac{x^2}{9} \Delta x \text{ cm}^3.$$

Taking a limit as $\Delta x \to 0$, we get

$$\text{Total volume } = \lim_{\Delta x \to 0} \sum \pi \frac{x^2}{9} \Delta x = \int_0^6 \pi \frac{x^2}{9}\, dx \text{ cm}^3.$$

Evaluating, we get

$$\text{Total volume } = \frac{\pi}{9} \frac{x^3}{3} \bigg|_0^6 = \frac{\pi}{9} \cdot \frac{6^3}{3} = 8\pi \text{ cm}^3.$$

Check: The volume of the cone can also be calculated using the formula $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 2^2 \cdot 6 = 8\pi$ cm^3.

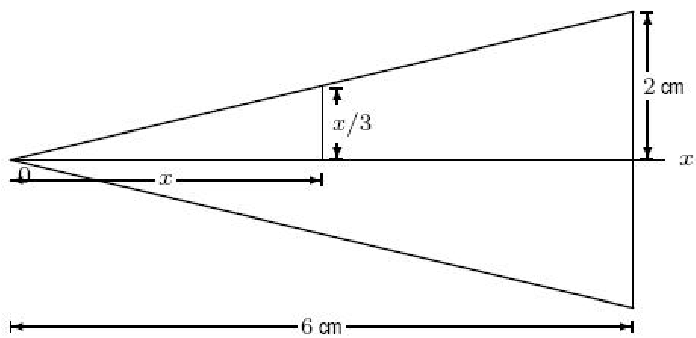

Figure 8.1

11. Each slice is a circular disk. From Figure 8.2, we see that the radius at height y is $r = \frac{2}{5}y$ cm. Thus

$$\text{Volume of disk } \approx \pi r^2 \Delta y = \pi \left(\frac{2}{5}y\right)^2 \Delta y = \frac{4}{25}\pi y^2 \Delta y \text{ cm}^3.$$

Summing over all disks, we have

$$\text{Total volume } \approx \sum \frac{4\pi}{25} y^2 \Delta y \text{ cm}^3.$$

Taking the limit as $\Delta y \to 0$, we get

$$\text{Total volume } = \lim_{\Delta y \to 0} \sum \frac{4\pi}{25} y^2 \Delta y = \int_0^5 \frac{4\pi}{25} y^2\, dy \text{ cm}^3.$$

Evaluating gives

$$\text{Total volume } = \frac{4\pi}{25} \frac{y^3}{3} \bigg|_0^5 = \frac{20}{3}\pi \text{ cm}^3.$$

Check: The volume of the cone can also be calculated using the formula $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3} 2^2 \cdot 5 = \frac{20}{3}\pi$ cm^3.

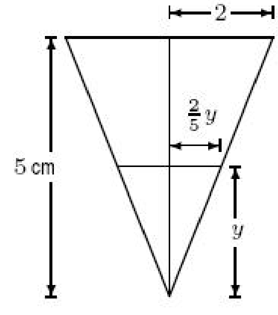

Figure 8.2

21. Cone with height 6 and radius 3. See Figure 8.12.

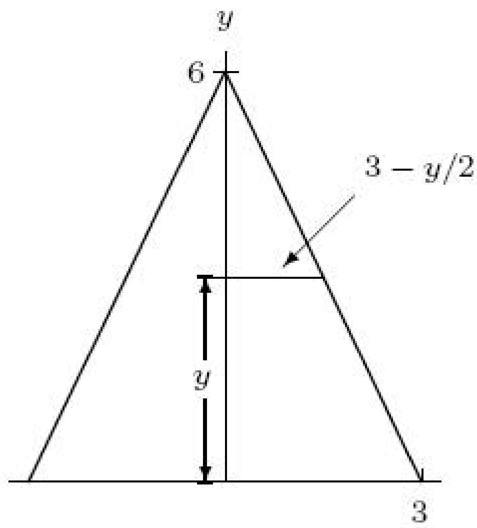

Figure 8.12