Calculus 112 Practice Problems

Section 8.2 Problems #1, #21, #22, #35

1. The volume is given by

$$V = \int_0^1 \pi y^2\,dx = \int_0^1 \pi x^4\,dx = \pi \frac{x^5}{5}\Big|_0^1 = \frac{\pi}{5}.$$

21. The two functions intersect at $(0,0)$ and $(8,2)$. We slice the volume with planes perpendicular to the line $x = 9$. This divides the solid into thin washers with volume

$$\text{Volume of slice } = \pi r_{out}^2 \Delta y - \pi r_{in}^2 \Delta y.$$

The outer radius is the horizontal distance from the line $x = 9$ to the curve $x = y^3$, so $r_{out} = 9 - y^3$. Similarly, the inner radius is the horizontal distance from the line $x = 9$ to the curve $x = 4y$, so $r_{in} = 9 - 4y$. Integrating from $y = 0$ to $y = 2$ we have

$$V = \int_0^2 \left[\pi(9 - y^3)^2 - \pi(9 - 4y)^2\right] dy.$$

22. The two functions intersect at $(0,0)$ and $(8,2)$. We slice the volume with planes perpendicular to the line $y = 3$. This divides the solid into thin washers with

$$\text{Volume of slice } = \pi r_{out}^2 \Delta x - \pi r_{in}^2 \Delta x.$$

The outer radius is the vertical distance from the line $y = 3$ to the curve $y = \frac{1}{4}x$, so $r_{out} = 3 - \frac{1}{4}x$. Similarly, the inner radius is the vertical distance from the line $y = 3$ to the curve $y = \sqrt[3]{x}$, so $r_{in} = 3 - \sqrt[3]{x}$. Integrating from $x = 0$ to $x = 8$ we have

$$V = \int_0^8 \left[\pi(3 - \frac{1}{4}x)^2 - \pi(3 - \sqrt[3]{x})^2\right] dx.$$

35. Slicing perpendicularly to the x-axis gives squares whose thickness is Δx and whose side is $1 - y = 1 - x^2$. See Figure 8.26. Thus

$$\text{Volume of square slice } \approx (1 - x^2)^2 \Delta x = (1 - 2x^2 + x^4)\,\Delta x.$$

$$\text{Volume of solid } = \int_0^1 (1 - 2x^2 + x^4)\,dx = x - \frac{2}{3}x^3 + \frac{x^5}{5}\Big|_0^1 = \frac{8}{15}.$$