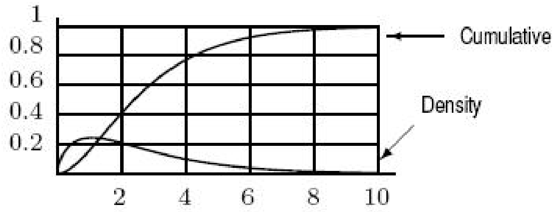
Calculus 112 Practice Problems

Section 8.7 Problems #16, #18, #19

16. (a) The two functions are shown below. The choice is based on the fact that the cumulative distribution does not decrease.
(b) The cumulative distribution levels off to 1, so the top mark on the vertical scale must be 1.



The total area under the density function must be 1. Since the area under the density function is about 2.5 boxes, each box must have area $1/2.5 = 0.4$. Since each box has a height of 0.2, the base must be 2.

18. (a) The percentage of calls lasting from 1 to 2 minutes is given by the integral

$$\int_1^2 p(x)\,dx \int_1^2 0.4e^{-0.4x}\,dx = e^{-0.4} - e^{-0.8} \approx 22.1\%.$$

(b) A similar calculation (changing the limits of integration) gives the percentage of calls lasting 1 minute or less as

$$\int_0^1 p(x)\,dx = \int_0^1 0.4e^{-0.4x}\,dx = 1 - e^{-0.4} \approx 33.0\%.$$

(c) The percentage of calls lasting 3 minutes or more is given by the improper integral

$$\int_3^\infty p(x)\,dx = \lim_{b\to\infty}\int_3^b 0.4e^{-0.4x}\,dx = \lim_{b\to\infty}(e^{-1.2} - e^{-0.4b}) = e^{-1.2} \approx 30.1\%.$$

(d) The cumulative distribution function is the integral of the probability density; thus,

$$C(h) = \int_0^h p(x)\,dx = \int_0^h 0.4e^{-0.4x}\,dx = 1 - e^{-0.4h}.$$

19. (a) The fraction of students passing is given by the area under the curve from 2 to 4 divided by the total area under the curve. This appears to be about $\frac{2}{3}$.
(b) The fraction with honor grades corresponds to the area under the curve from 3 to 4 divided by the total area. This is about $\frac{1}{3}$.
(c) The peak around 2 probably exists because many students work to get just a passing grade.
(d)