Calculus 112 Practice Problems

Section 8.8 Problems #4, #5, #7

4. (a) Since $d(e^{-ct})/dt = ce^{-ct}$, we have

$$c\int_0^6 e^{-ct}dt = -e^{-ct}\Big|_0^6 = 1 - e^{-6c} = 0.1,$$

so

$$c = -\frac{1}{6}\ln 0.9 \approx 0.0176.$$

(b) Similarly, with $c = 0.0176$, we have

$$\begin{aligned} c\int_6^{12} e^{-ct}dt &= -e^{-ct}\Big|_6^{12} \\ &= e^{-6c} - e^{-12c} = 0.9 - 0.81 = 0.09, \end{aligned}$$

so the probability is 9%.

5. (a) We can find the proportion of students by integrating the density $p(x)$ between $x = 1.5$ and $x = 2$:

$$\begin{aligned} P(2) - P(1.5) &= \int_{1.5}^2 \frac{x^3}{4}\,dx \\ &= \frac{x^4}{16}\Big|_{1.5}^2 \\ &= \frac{(2)^4}{16} - \frac{(1.5)^4}{16} = 0.684, \end{aligned}$$

so that the proportion is $0.684 : 1$ or 68.4%.

(b) We find the mean by integrating x times the density over the relevant range:

$$\begin{aligned} \text{Mean} &= \int_0^2 x\left(\frac{x^3}{4}\right)\,dx \\ &= \int_0^2 \frac{x^4}{4}\,dx \\ &= \frac{x^5}{20}\Big|_0^2 \\ &= \frac{2^5}{20} = 1.6 \text{ hours}. \end{aligned}$$

(c) The median will be the time T such that exactly half of the students are finished by time T, or in other words

$$\begin{aligned} \frac{1}{2} &= \int_0^T \frac{x^3}{4}\,dx \\ \frac{1}{2} &= \frac{x^4}{16}\Big|_0^T \\ \frac{1}{2} &= \frac{T^4}{16} \\ T &= \sqrt[4]{8} = 1.682 \text{ hours}. \end{aligned}$$

7. (a) The cumulative distribution function

$$P(t) = \int_0^t p(x)dx = \text{Area under graph of density function } p(x) \text{ for } 0 \le x \le t$$
$$= \text{Fraction of population who survive } t \text{ years or less after treatment}$$
$$= \text{Fraction of population who survive up to } t \text{ years after treatment.}$$

(b) The probability that a randomly selected person survives for at least t years is the probability that he lives t years or longer, so

$$S(t) = \int_t^\infty p(x)\,dx = \lim_{b\to\infty}\int_t^b Ce^{-Ct}\,dx$$
$$= \lim_{b\to\infty} -e^{-Ct}\Big|_t^b = \lim_{b\to\infty} -e^{-Cb} - (-e^{-Ct}) = e^{-Ct},$$

or equivalently,

$$S(t) = 1 - \int_0^t p(x)\,dx = 1 - \int_0^t Ce^{-Ct}\,dx = 1 + e^{-Ct}\Big|_0^t = 1 + (e^{-Ct} - 1) = e^{-Ct}.$$

(c) The probability of surviving at least two years is

$$S(2) = e^{-C(2)} = 0.70$$

so

$$\ln e^{-C(2)} = \ln 0.70$$
$$-2C = \ln 0.7$$
$$C = -\frac{1}{2}\ln 0.7 \approx 0.178.$$