Calculus 112 Practice Problems

Section 9.1 Problems #20, #21, #22, #29, #30, #41, #43

20. We have:

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)=0.$$

The terms of the sequence alternate in sign, but they approach 0, so the sequence converges to 0.

21. Since the exponential function 2^n dominates the power function n^3 as $n\to\infty$, the series diverges.

22. As n increases, the term $2n$ is much larger in magnitude than $(-1)^n 5$ and the term $4n$ is much larger in magnitude than $(-1)^n 3$. Thus dividing the numerator and denominator by n and using the fact that $\lim_{n\to\infty} 1/n = 0$, we have

$$\lim_{n\to\infty}\frac{2n+(-1)^n 5}{4n-(-1)^n 3}=\lim_{n\to\infty}\frac{2+(-1)^n 5/n}{4-(-1)^n 3/n}=\frac{1}{2}.$$

Thus, the sequence converges to $1/2$.

29. We have $s_2 = s_1 + 2 = 3$ and $s_3 = s_2 + 3 = 6$. Continuing, we get

$$1,\ 3,\ 6,\ 10,\ 15,\ 21.$$

30. We have $s_2 = s_1 + 1/2 = 0 + (1/2)^1 = 1/2$ and $s_3 = s_2 + (1/2)^2 = 1/2 + 1/4 = 3/4$. Continuing, we get

$$0,\ \frac{1}{2},\ \frac{3}{4},\ \frac{7}{8},\ \frac{15}{16},\ \frac{31}{32}.$$

41. The differences between consecutive terms are 4, 9, 16, 25, so, for example, $s_2 = s_1 + 4$ and $s_3 = s_2 + 9$. Thus, a possible recursive definition is $s_n = s_{n-1} + n^2$ for $n > 1$ and $s_1 = 1$.

43. The numerator and denominator of each term are related to the numerator and denominator of the previous term. The denominator is the previous numerator and the numerator is the sum of the previous numerator and previous denominator. For example,

$$\frac{5}{3}=\frac{2+3}{3}\text{ and }\frac{8}{5}=\frac{3+5}{5}.$$