Calculus 112 Practice Problems

Section 9.3 Problems #14, #16

14. Since the terms in the series are positive and decreasing, we can use the integral test. We calculate the corresponding improper integral:

$$\int_0^\infty \frac{4}{2x+1}\,dx = \lim_{b\to\infty}\int_0^b \frac{4}{2x+1}\,dx = \lim_{b\to\infty} 2\ln(2x+1)\Big|_0^b = \lim_{b\to\infty} 2\ln(2b+1).$$

Since the limit does not exist, the integral diverges, so the series $\displaystyle\sum_{n=0}^{\infty}\frac{4}{2n+1}$ diverges.

16. Since the terms in the series are positive and decreasing, we can use the integral test. We calculate the corresponding improper integral using the substitution $w = x^2$

$$\int_0^\infty \frac{2x}{1+x^4}\,dx = \lim_{b\to\infty}\int_0^b \frac{2x}{1+x^4}\,dx = \lim_{b\to\infty} \arctan x^2\Big|_0^b = \lim_{b\to\infty} \arctan b^2 = \frac{\pi}{2}.$$

Since the limit exists, the integral converges, so the series $\displaystyle\sum_{n=0}^{\infty}\frac{2n}{1+n^4}$ converges.