Calculus 112 Practice Problems

Section 9.5 Problems #7, #23, #24, #27

7. The general term can be written as $\dfrac{(-1)^k(x-1)^{2k}}{(2k)!}$ for $k \geq 0$. Other answers are possible.

23. Let $C_n = 2^n/n$. Then replacing n by $n+1$ gives $C_{n+1} = 2^{n+1}/(n+1)$. Using the ratio test, we have

$$\frac{|a_{n+1}|}{|a_n|} = |x|\frac{|C_{n+1}|}{|C_n|} = |x|\frac{2^{n+1}/(n+1)}{2^n/n} = |x|\frac{2^{n+1}}{n+1}\cdot\frac{n}{2^n} = 2|x|\left(\frac{n}{n+1}\right).$$

Thus

$$\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|} = 2|x|.$$

The radius of convergence is $R = 1/2$.

For $x = 1/2$ the series becomes the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n}$ which diverges.

For $x = -1/2$ the series becomes the alternating series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ which converges. See Example 8 on page 486.

$n=1$

24. We use the ratio test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{3^{n+1}}\cdot\frac{3^n}{x^n}\right| = \frac{|x|}{3}.$$

Since $|x|/3 < 1$ when $|x| < 3$, the radius of convergence is 3 and the series converges for $-3 < x < 3$.
We check the endpoints:

$$x = 3: \quad \sum_{n=0}^{\infty}\frac{x^n}{3^n} = \sum_{n=0}^{\infty}\frac{3^n}{3^n} = \sum_{n=0}^{\infty}1^n \quad \text{which diverges.}$$

$$x = -3: \quad \sum_{n=0}^{\infty}\frac{x^n}{3^n} = \sum_{n=0}^{\infty}\frac{(-3)^n}{3^n} = \sum_{n=0}^{\infty}(-1)^n \quad \text{which diverges.}$$

The series diverges at both the endpoints, so the interval of convergence is $-3 < x < 3$.

27. We use the ratio test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{n+1}(x-5)^{n+1}}{2^{n+1}(n+1)^2} \cdot \frac{2^n n^2}{(-1)^n(x-5)^n}\right| = \left(\frac{n}{n+1}\right)^2 \cdot \frac{|x-5|}{2}.$$

Since $n/(n+1) \to 1$ as $n \to \infty$, we have

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-5|}{2}.$$

We have $|x-5|/2 < 1$ when $|x-5| < 2$. The radius of convergence is 2 and the series converges for $3 < x < 7$.

We check the endpoints. For $x = 3$, we have

$$\sum_{n=1}^{\infty}\frac{(-1)^n(x-5)^n}{2^n n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n(3-5)^n}{2^n n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n(-2)^n}{2^n n^2} = \sum_{n=1}^{\infty}\frac{1}{n^2}.$$

This is a p-series with $p = 2$ and it converges. For $x = 7$, we have

$$\sum_{n=1}^{\infty}\frac{(-1)^n(x-5)^n}{2^n n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n(7-5)^n}{2^n n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n 2^n}{2^n n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}.$$

Since $\sum \frac{1}{n^2}$ converges, the alternating series $\sum \frac{(-1)^n}{n^2}$ also converges. The series converges at both its endpoints, so the interval of convergence is $3 \le x \le 7$.