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An Insight Into the Importance of Wrist Torque in Driving the Golfball: A Simulation Study

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The purpose of this study was to examine whether, in theory, the clubhead speed at impact could be increased by an optimally timed wrist torque, without jeopardizing the desired club position at impact. A 2-D, three-segment model comprising torso, left arm, and golfclub was used to model the downward phase of the golf swing. Torque generators that adhered to the activation and force-velocity properties of muscle were inserted at the proximal end of each segment. Separate simulations were performed, with the wrist joint generator enabled then disabled. The results from these simulations showed that significant gains in clubhead speed (≈9 %) could be achieved if an active wrist torque was applied to the club during the latter stages of the downswing. For a swing that produced a clubhead speed of 44 m/s (≈99 mph), the optimal timing for the activation of wrist torque occurred when the arm segment was approximately 30° below a horizontal line through the shoulder joint. The optimal activation time for the joint generators was very much dependant on the shape of the torque profiles. The optimization process confirmed that maximum clubhead speed was achieved when the torque generators commenced in sequential order from proximal to distal.

Key Words: simulation, optimization, golf kinematics, wrist torque

Introduction

The speed of the clubhead at impact is the principal factor that determines the distance that a golfball will travel. Clubhead speed is known to be a function of the sequential segment velocities of the chain link that makes up the golf swing (Herring & Chapman, 1992). The wrist joint, being the most distal anatomical joint in this chain, would be expected to play a role in the development of the final clubhead speed. Over the years, golfers have debated whether the action of the wrists should be passive or active in the releasing of the clubhead prior to impact. Most golfers have an intuitive belief that the addition of a properly timed muscular torque at the wrist joint will increase clubhead speed at impact. However, a number of pundits, including the legendary Bobby Jones,

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believe the opposite is true. Jones (1966) stated that during the swing the club "free-wheeled" through the ball. Williams (1967), who worked from a stroboscopic photograph of Bobby Jones swing, likewise concluded that the uncocking torque applied by the hands was negligible. Jorgensen (1994), using a computer simulation of a two segment planar model of the golf swing, provided insight into this freewheeling theory of the golf swing. His simulation work revealed, paradoxically, that anything the golfer does with his wrists during the downswing to decrease the wrist-cock angle results in less clubhead speed at impact than if the golfer allows the wrist joint to open naturally. While Jorgensen (1994) reported that the early onset of muscular wrist torque during the downswing was ill advised, his simulation work revealed that clubhead speed might be increased marginally (0.7%) if the torque was delayed until 0.07 s prior to impact. However, he cautioned that the effectiveness of this delayed wrist torque was very sensitive to its duration time; longer or shorter duration times produced clubhead speeds less than maximal.

The simulation work by Jorgensen, while very instructive, was based on a two-segment golf swing system, comprising an arm and a club segment, with torque generators inserted at the wrist and shoulder joints. The shoulder and wrist torque generators in his model, when activated, were constant in magnitude, with no allowance made for the force-velocity or activation properties of muscle. These modeling limitations suggest that the potential role of wrist torque in the golf swing may still not be fully understood. The purpose of this paper was to re-examine the question as to whether, in theory, a properly timed wrist torque during the downswing can significantly improve clubhead speed without jeopardizing the desired club position at impact.

Methods

The golfer was modeled as a three-segment, two dimensional (2-D), linked system with the golfclub, arm, and torso segments moving in a plane tilted 60° to the ground (Figure 1). The assumption of planar movement of these segments during the downward swing is well supported in golf literature (Cochran & Stobbs, 1968; Jorgensen, 1994). The golfclub was modeled as a rigid segment which is consistent with the conclusion of Milne and Davis (1992) that, contrary to popular belief, shaft bending flexibility plays only a minor dynamic role in the golf swing. For the purposes of the 2-D representation, the torso was collapsed along its longitudinal axis so that it lay in the movement plane as a rigid rod with a length equal to the distance from the sternal notch of the sternum to the glenoid fossa of the scapula. Torque generators were inserted at the proximal end of each segment and provided the model with the capability of adding energy to the system. The torque generators used in the simulation were programmed to be constrained by the activation rate and force-velocity properties of human muscle. The force-length property of muscle was expected to play a second-order role in the outcome of the performance (Caldwell, 1995) and, as such, was not included in the simulation model. The activation rate and force-velocity properties associated with human muscle were implemented using the calculated instantaneous isometric torque predicted from a linearized Hill model structure (Niku & Henderson, 1985; Sprigings, 1986) as input to the force-velocity approach described by Alexander (1990).

$$\mathrm{T} = \mathrm{T}_{\max}\left(1 - e^{-t/\tau}\right)\frac{(\omega_{\max} - \omega)}{(\omega_{\max} + \Gamma\omega)} \tag{1}$$

In equation 1, T is the instantaneous value produced by a torque generator; T_{max} is the maximum isometric torque of the torque generator; ω_{max} is the maximum angular velocity of the associated joint; ω is the instantaneous joint angular velocity; Γ is a shape factor controlling the curvature of the torque/velocity relationship; t is the elapsed time from initial torque activation; and τ is the activation time constant (Sprigings, 1986; Pandy, Zajac, Sim, & Levine, 1990). For the present study, T_{max} was set at 180, 120, 60 Nm for the spine, shoulder, and wrist, respectively (Neal, Burko, Sprigings, & Landeo, 1999); ω_{max} was set at 20, 30, 60 rad $\cdot$ s^{-1} for the spine, shoulder, and wrist, respectively (Neal et al., 1999); τ was set at 40 ms (Sprigings, 1986); and Γ was assigned a value of 3.0 (Alexander, 1990).

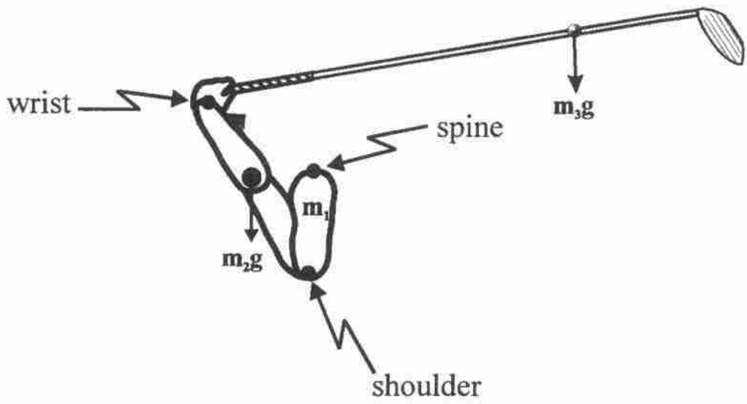

Figure 1 — Three-segment model comprising torso, left arm, and club, positioned at the top of the backswing.

Passive protective linear-elastic torque elements were installed at each joint and were only activated if the anatomical limits of a particular joint were in danger of being exceeded during the simulation. Parameter values for segment length, moment of inertia, and mass for a representative golfer with a body mass of 80 kg and a standing height of 1.83 m were calculated using the values of de Leva (1996). Parameter values for a standard driver, 43.5 in. in length, were taken from the work of Cochran and Stobbs (1968).

The equations of motion for the three-segment system were written using a Newtonian formulation in combination with the known equations of constraint for a system linked with pin joints (Sprigings, Lanovaz, Watson, & Russell, 1998). The torso segment was treated as a special case in that gravitational force was prevented from supplying rotational motion to the segment. We reasoned that during the golf swing, the torso segment rotates about the spine, and since the spine is a line of symmetry for this segment, gravitational torque cannot assist rotation about this axis. The value used for the moment of inertia of the model's torso segment about its proximal end was that of the anatomical torso's moment of inertia about its longitudinal axis. A fifth-order Runge-Kutta-Fehlberg algorithm (Burden, Faires, & Reynolds, 1981) with variable step size was programmed and used to drive the simulation model.

The simulation process commenced with the assumption that the golfer had just completed his back swing and was just about to commence his down swing. It was assumed that at time zero the golfer's torso segment was rotated 90° clockwise (top view) from the address position, with the arm and club segments positioned 60° and 10°, respectively, above a horizontal line through their proximal end, which is a typical configuration for an elite golfer (Yun, 1996; Figure 1). The acute 70° of wrist-cock angle that corresponds to this starting configuration takes into account the club's inertial effects that are observed for a real player during the dynamic transition from the backward to forward swing.

The optimization scheme employed a single activation muscular control strategy where the onset of voluntary torque at each joint was controlled separately. The time of onset, as well as the length of time that the joint torques acted, provided six control variables for the optimization. The optimization search engine was based on Powell's algorithm (Press, Teukolsky, Vetterling, & Flannery, 1992). The objective function was composed of the clubhead speed at impact, along with penalty variables that reflected inappropriate behavior by the model during the simulated golf swing. For example the position of the club's shaft at impact was constrained using a penalty variable to be within $\pm 0.5°$ of the vertical position. This impact position constraint is consistent with the observation made by internationally acclaimed professional golf instructor, Jim McLean, that the greatest drivers of the modern era (Nicklaus, Norman, Hogan, Nelson, Snead, Price, Woods, Lietzke, Peete, Sutton) when viewed face on, all had their clubshaft vertical at impact (McLean, 1999). In the optimization scheme, the clubhead speed was expressed as a negative valued penalty variable so that its minimization in the optimization scheme would actually reflect a maximum. The sum of the accrued penalty variables served as the objective function, which was minimized by varying the values of the six control parameters that regulated the onset and duration of the muscular torque activation strategy.

The simulation sequence was terminated when the clubhead reached a position 20 cm horizontally past the proximal end of the torso segment, or if the simulation time exceeded 0.9 s. Although a small time-step interval of .002 s was used during the simulation runs, the exact time of impact was determined by means of interpolation. To reduce the chance of arriving at a local instead of a global minimum during the optimization process, 200 randomly generated starting conditions for each of the six control variables were examined for each optimization trial. The set of "best" starting conditions, as determined by the magnitude of the penalty summation that accrued during each simulation, was then used as the starting conditions for the POWELL optimization process. At the termination of the POWELL optimization process, the optimized set of control variables was stored in memory. This entire procedure was repeated 50 times, with the "best" set of optimized control variables being saved as a permanent file. The magnitudes of all torque generators were set to zero until activated by the optimization process.

Three simulation conditions for the downward phase of the golf swing were optimized. The first simulation condition (SIM-1) provided for the presence of voluntary wrist torque during the optimization search for maximum clubhead speed at impact. The second condition (SIM-2) prevented any voluntary wrist torque being used during the optimization process, which effectively reduced the wrist to a free hinge during the downward swing. A third simulation (SIM-3) was performed where the force-velocity property of muscle was not included in the joint torque generators. The purpose of this third simulation condition was to examine the effect that not including force-velocity property might have on the segment timing reported by earlier simulation studies.

A qualitative validation test was performed on the simulation results by comparing the corresponding image sequences for the three-segment model with a real-life photographic sequence of an elite professional golfer during his downswing.

Results

For the first simulation condition (SIM-1), which permitted a voluntary wrist torque to be present if it improved clubhead speed at impact, the maximum horizontal clubhead speed at impact was 44.0 m/s ($\approx$99 mph; Figure 2). The onset of voluntary muscular torque at the joints (Figure 3) demonstrated a proximal to distal pattern, with the torso segment's torque generator being turned on immediately, followed by the shoulder torque for the arm segment 0.148 s later, followed finally by the torque at the wrist joint 0.138 s after that. The total time of the downward swing to impact was 0.380 s, which is comparable to a value of 0.34 s measured from video for professional golfer, Nick Faldo, whose club speed at impact was approximately 5 m/s faster. For the SIM-1 simulation condition, removing the constraint that forced the clubshaft to be in the vertical position at impact produced a slight improvement in clubhead speed (≈2%) but at the expense of good clubface alignment at impact.

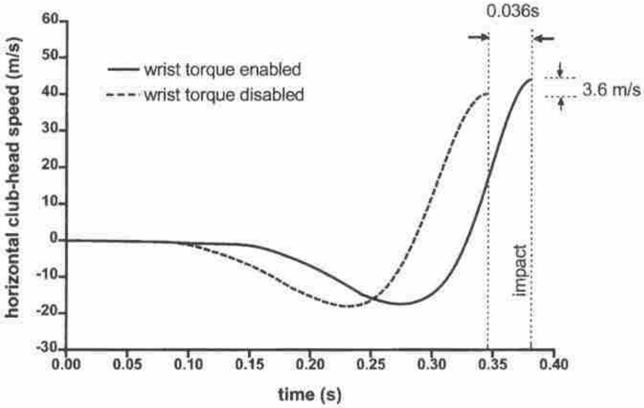

Figure 2 — Velocity of the clubhead throughout the downswing under two conditions: (a) wrist torque generator enabled; and (b) wrist torque generator disabled.

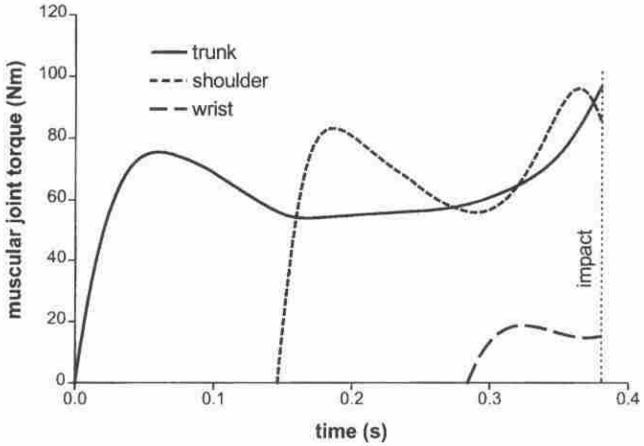

Figure 3 — Profiles of the joint torque histories when the wrist torque generator was enabled (SIM-1). The clubhead reaches a speed of 44.0 m/s (≈99 mph).

Under virtually static conditions at the initiation of the downswing, the muscular torque generated by the torso reached a value of approximately 75 Nm before decreasing as a result of the force-velocity properties of the torque generator as the torso picked up rotational speed (Figure 3). A similar pattern was observed for the early phase of the torque generator at the shoulder, where it reached a value of 83 Nm before momentarily decreasing as a result of the force-velocity properties of the generator that were attenuated by the arm's increasing angular velocity relative to the torso. As impact neared, the muscular torques generated by both the torso and shoulder increased again as a result of the torque generator at the wrist joint being activated. The reason for this behavior was that the activation of the wrist torque generator increased the angular velocity of the club segment, which in turn decreased the angular velocities of the arm and torso segments. This reduction of the relative angular velocities of the torso and arm segments, via segment interactions, enabled the output of their torque generators to increase as impact neared. The final brief reduction in torque output at the shoulder joint is attributed to a brief increase in the arm's relative angular velocity that coincides with a slight decrease in angular velocity of the wrist joint just before impact.

The optimized simulation, SIM-1 (Figure 4A), revealed that the active wrist torque commenced shortly after the natural uncocking of the wrist joint had begun. Thus the wrist torque was not responsible for initiating the uncocking of the wrist joint but was employed to augment the naturally occurring wrist action induced by the centrifugal pull of the club. This active uncocking of the wrist joint using muscular torque was delayed until the arm segment was approximately 30° below a horizontal line through the shoulder joint.

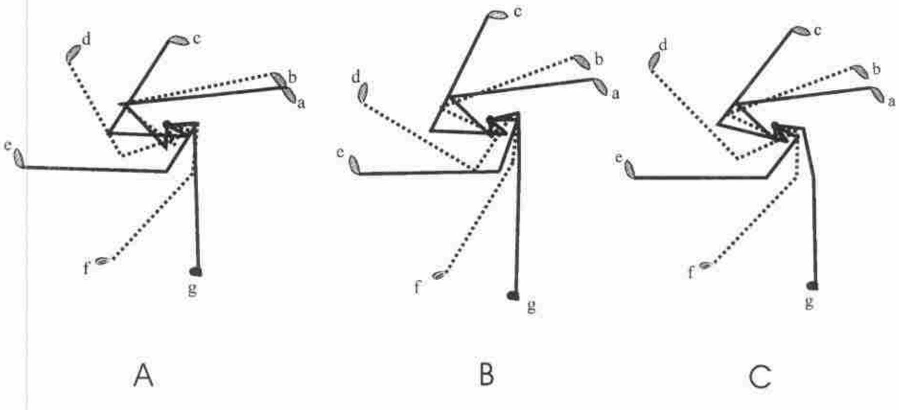

Figure 4 — A. Sequential pattern of the simulated golf swing under the conditions of SIM-1. Position "c" corresponds to the start of the uncocking of the wrist as a result of the centrifugal pull on the club. Position "d" corresponds to the start of active muscular wrist torque being generated during the swing. B. Sequential pattern of the real life swing of professional golfer, Nick Faldo. C. Sequential pattern of the simulated golf swing under the conditions of SIM-2. Position "c" corresponds to the start of the uncocking of the wrist as a result of the centrifugal pull on the club.

The sensitivity of the simulation to the timing of the wrist torque was examined by advancing and delaying the onset of wrist torque by 50 ms from that found to be optimal. The results revealed that activating wrist torque 50 ms late reduced the clubhead speed at impact by 4.6%, as compared to a reduction of 2.0% when the wrist torque was activated 50 ms early.

A visual comparison between the image sequence of SIM-1 (Figure 4A) and that of a real life elite professional golfer, Nick Faldo (Figure 4B), revealed a marked similarity in the starting and end positions. The only apparent difference between the two sequences appears in the greater delay of the wrist uncocking during the downswing for the elite golfer. This can be attributed to the greater rotational speed of the torso exhibited by the professional golfer as he completed the downward swing in less time.

The second simulation condition (SIM-2) successfully showed that it is possible to reach the desired impact position with the clubshaft vertical without using muscular wrist torque during the downward swing (Figure 4C). The maximum horizontal clubhead speed reached in SIM-2 was 40.4 m/s (≈91 mph; Figure 2), which is approximately 3.6 m/s (≈8 mph) slower than that achieved using SIM-1, where wrist torque was permitted. As in SIM-1, the onset of the voluntary muscular torque for the torso and the arm segments in SIM-2 was proximal to distal in nature, with the arm's shoulder torque generator being activated 0.080 s after the onset of the torso's torque (Figure 5). With the absence of voluntary wrist torque, the time of the downward swing (0.344 s) was slightly shorter by 0.036 s than it was when wrist torque was employed. This difference was somewhat surprising considering that the final velocity reached by the clubhead was significantly greater when wrist torque was applied. The explanation for this finding can be traced to the increased angular displacement of the torso segment observed in SIM-1. By increasing the angular displacement of the trunk, the arm's rotation relative to the trunk was delayed which, in turn, delayed the uncocking of the wrist (Figure 6). The net result was a higher speed of the clubhead at impact for SIM-1 compared to SIM-2, but developed over a longer time period as a result of the greater use of torso rotation.

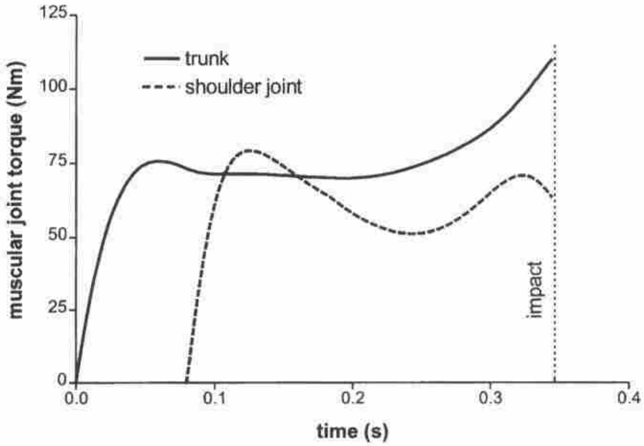

Figure 5 — Profiles of the joint torque histories when the wrist torque generator was disabled (SIM-2). The clubhead reaches a speed of 40.4 m/s (≈91 mph).

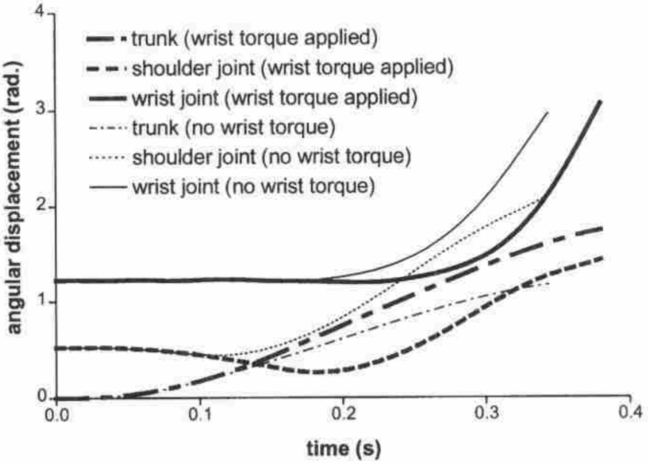

Figure 6 — Angular displacements of the joints of the three segments under two conditions: (a) wrist torque generator enabled; and (b) wrist torque generator disabled.

The results of the third simulation condition (SIM-3), in which the force-velocity property of muscle was removed from the torque generators, revealed that a significantly higher clubhead speed (≈57 m/s, or 128 mph) could be reached at impact, in a shorter period of time (0.248 s), even while constrained by the same upper torque limits that were imposed on SIM-1. However, the asymptotic shape of the associated muscular torque profiles (Figure 7) are unrealistic when compared to the shape of the muscular torque profiles for real golfers that were determined by means of inverse dynamics (Neal et al., 1999). Removing the force-velocity constraint from the simulation model had the effect of activating the wrist torque later in the simulation. Specifically, for SIM-3, the simulated muscular wrist torque was activated when the arm was approximately 60° below a horizontal line projected through the shoulder joint, as compared to 30° when the force-velocity property was incorporated into the model.

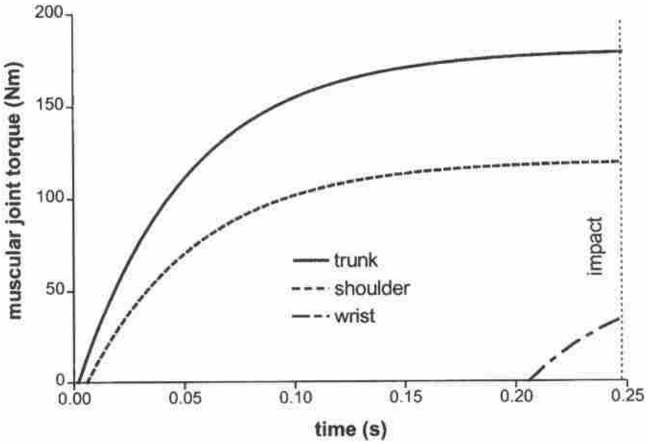

Figure 7 — Profiles of the torques produced by the joint generators when the force-velocity property of muscle was not incorporated into the model. The clubhead reaches a speed of 57 m/s (128 mph).

Discussion

Every model is an approximation to the truth, with many variables being neglected that are judged or calculated to be of minor importance to the conclusions reached (Hubbard & Trinkle, 1984). The necessary level of complexity that one builds into a simulation model depends on the question under study. Alexander (1990, 1992) and Hubbard (1993) caution against the use of complex mathematical models whose results become impossible to interpret because of the large number of inextricably intertwined independent variables. From the general agreement of the results with those of a recognized elite professional golfer, it appears that a simple 2-D, three-segment model, using a single activation control strategy with specialized torque generators at the proximal ends, has sufficient modeling detail to predict the optimal timing necessary to achieve maximum clubhead speed at impact.

In recent years there has been conjecture (Jorgensen, 1994) as to whether a good golfer should actively uncock his wrists during the later stages of the golf swing, or whether the release of the clubhead should be allowed to occur naturally as a consequence of the centrifugal pull of the clubhead itself on the arm segment. From our simulation results it is clearly evident that significant gains in clubhead speed (≈9%) can be achieved if muscular wrist torque is employed during the later stages of the downward swing just prior to impact. Optimizing the timing of the torque generators used in the model required the use of a proximal to distal muscular activation pattern if maximal clubhead speed was to be achieved. The simulation results also supported the observation made by Jorgensen (1994) that any active muscular wrist torque must be delayed by the golfer until his/her arms are approximately 30° below a horizontal line through the shoulder joint. Any earlier activation of the muscular wrist torque resulted in a reduction in clubhead speed. Likewise, activation of the torque generators in an order different from proximal to distal resulted in a less than optimal performance as measured by clubhead speed. In fact, a simulation in which the joint torque generators were forced to turn on simultaneously at the start of the downswing produced a clubhead speed of only 30.5 m/s (≈68 mph). This value equates to an approximate 30% reduction in clubhead speed from that produced when the timing was optimal.

The SIM-2 simulation condition clearly showed that it is possible to reach the desired impact position with the golfclub without using muscular wrist torque during the downward swing. This result lends support to the contention of such notable golfers as Bobby Jones who felt that during the swing, the club "freewheeled" through the ball (Jones, 1966). However, the simulation results for SIM-1 clearly show that the use of an optimally timed muscular wrist torque during the final phase of the downswing can produce gains of up to 9% in clubhead speed at impact. The implications for hitting the ball further are clear, since this increase in clubhead speed would correspond to an increase in ball speed off the tee of 4.9 m/s (≈11 mph; Daish, 1972).

Prior to the current study, simulation studies of the golf swing have not incorporated the force-velocity property of muscle into their model's torque generators. Our results have shown that such an omission will adversely affect the measured optimal timing pattern of segment involvement. Without the constraint imposed by the force-velocity property of muscle, the shoulder and wrist torque generators will be activated earlier in the down swing to take advantage of the additional angular impulse that can be produced by torque output profiles that rise asymptotically to a maximum. The three-segment model used in our study was able to use realistic magnitudes of muscular torques to generate clubhead speeds that are reflective of good golfers. The maximum values for the torques generated by the torso, shoulder, and wrist joints during either SIM-1 and SIM-2 simulation conditions (Figures 3 & 5) were 109 Nm, 96 Nm, and 18.5 Nm, respectively, which agree favorably with the upper values of torque (110, 90, and 30 Nm) measured directly from a of a low handicap amateur golfer using inverse dynamics (Neal et al., 1999). In comparison, the peak torque values of 339 and 191 Nm reported by Campbell and Reid (1985) for the torso and arm segment generators in their 3 segment model appear to be unrealistically high for a golf swing. Similarly, constant torque values of 200 Nm for the shoulder joint, as used by Lampsa (1975), would appear to be well beyond any golfer's physical capabilities. Of course with a two-segment model, such as the one used by Lampsa, unrealistically high shoulder torques have to be employed in the simulation model if clubhead speeds at impact are to reach values known to be attainable by good golfers.

One of the obvious disadvantages of two-segment models is that they cannot examine the importance of the torso in generating clubhead speed. The results produced from our three-segment model clearly showed that, for optimal performance, it was the active counter-clockwise rotation of the torso that initiated the downward sequence of the swing. When an active wrist torque was employed by the model, the magnitude of angular displacement that the torso rotated through to impact, increased. It was this strong rotation of the torso segment that was observed to be directly linked to the delay in the uncocking of the wrists that good golfers seek during the downswing.

In conclusion, our simulations have shown that a properly timed wrist torque applied by the golfer to the club's handle can produce gains of up to 9% in clubhead speed. Our simulation results also show that while muscular wrist torque is desirable for maximizing clubhead speed at impact, it is not a necessary requirement for aligning the clubshaft to the desired vertical position for impact.

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