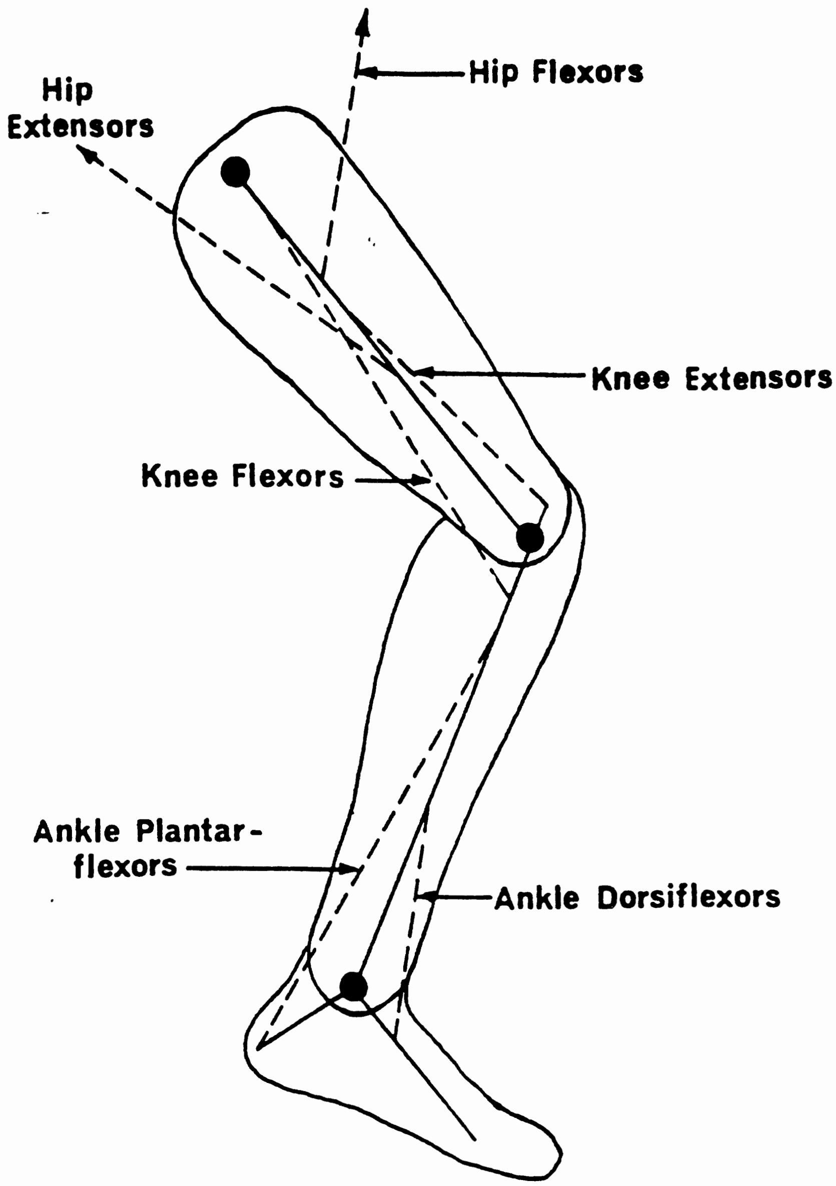


FIGURE 4-12. *Idealized Resultant Muscle Forces of the Lower Limb.*

To achieve consistency in the free body diagrams and to facilitate subsequent computer processing and data analysis, the following conventions have been adopted:

1. Lines of action of forces directed upward and to the right are positive while those directed downward or to the left are negative.
2. Rotation in a counterclockwise direction is considered positive and clockwise rotation negative.

3. All angles are expressed with respect to the right horizontal axis being zero.

4. Resultant force components and the resultant muscular torque at the proximal end of a segment are indicated as positive on the free body diagram while those at the distal end are negative.

5. The symbols employed include:

W(*i*)	segment weight
JX(*i*), JY(*i*)	*x* and *y* components of joint reaction force
F(*i*)	resultant muscle force

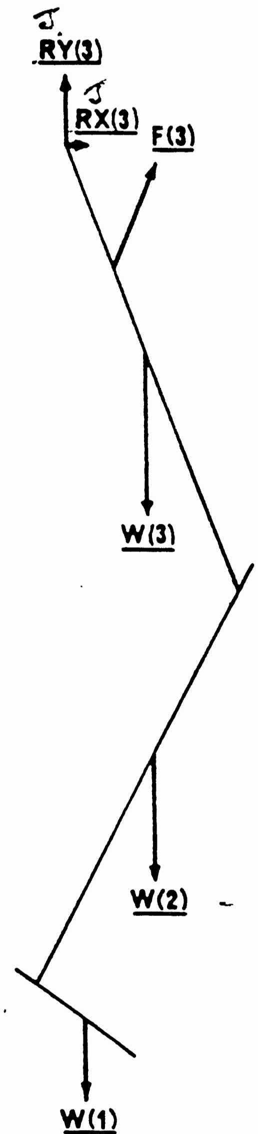


FIGURE 4-13. *Free Body Diagram of the Lower Limb.*

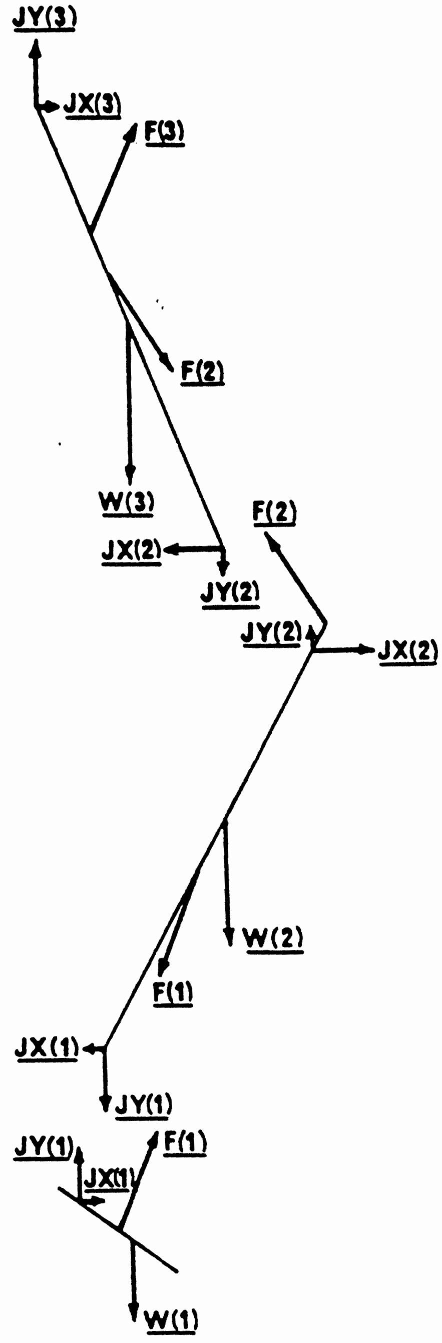


FIGURE 4-14. *Free Body Diagrams of the Three Limb Segments.*

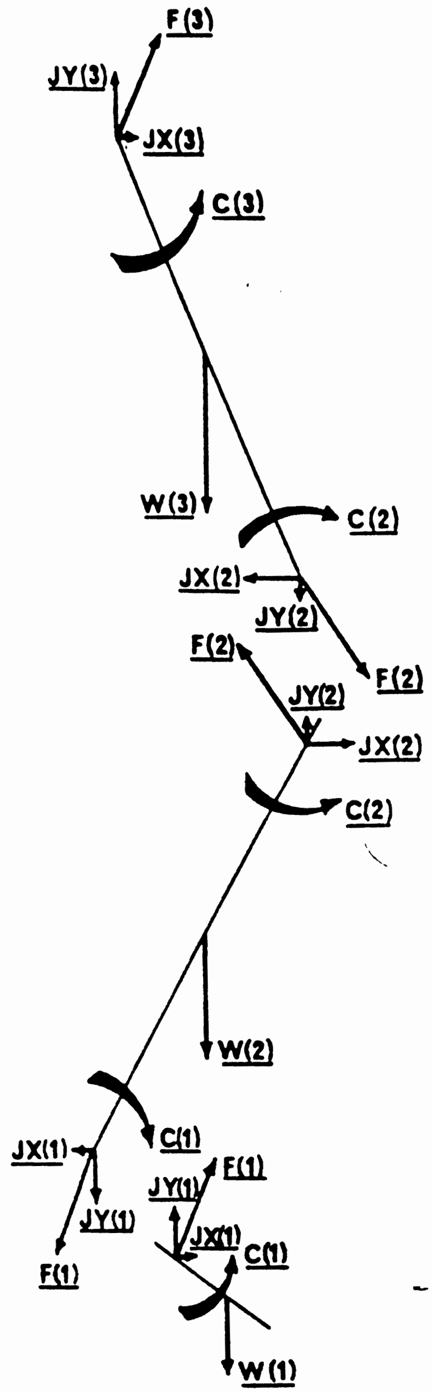


FIGURE 4-15. *Replacement of the Resultant Muscle Force with an Equivalent Force and Couple.*

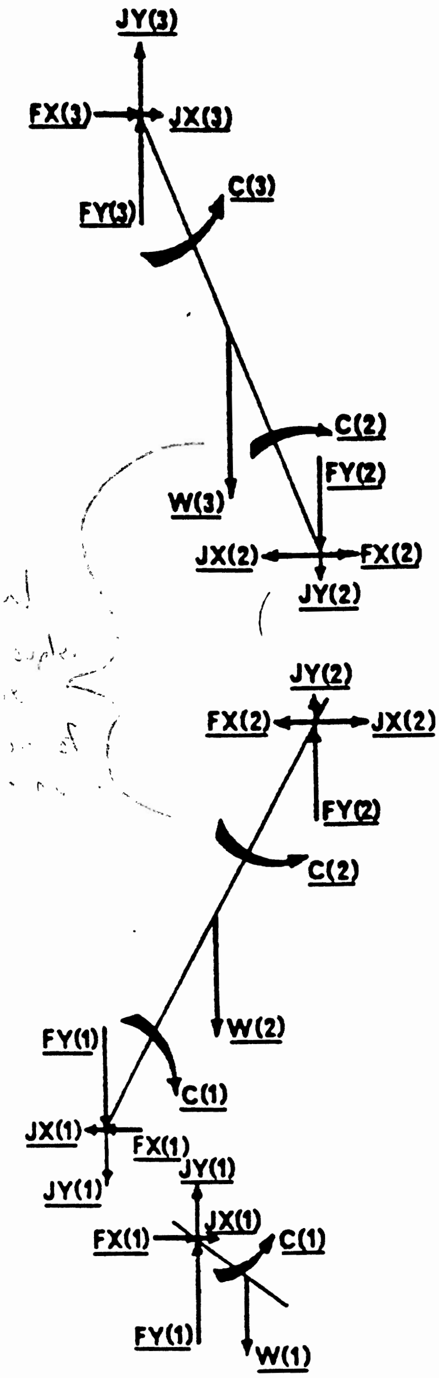


FIGURE 4-16. *Division of the Resultant Muscle Force into Horizontal and Vertical Components.*

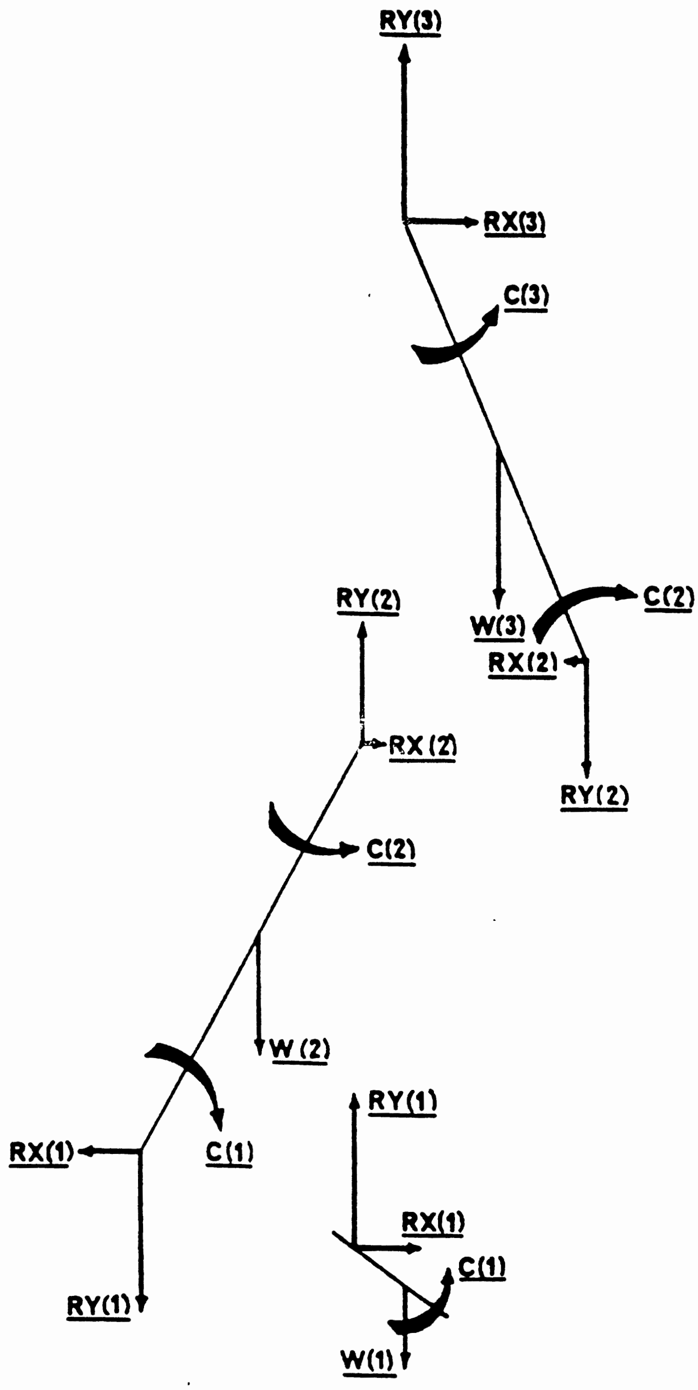


FIGURE 4-17. *Combination of Joint Reaction and Muscle Force Components into Resultant Force Components at the Joint.*

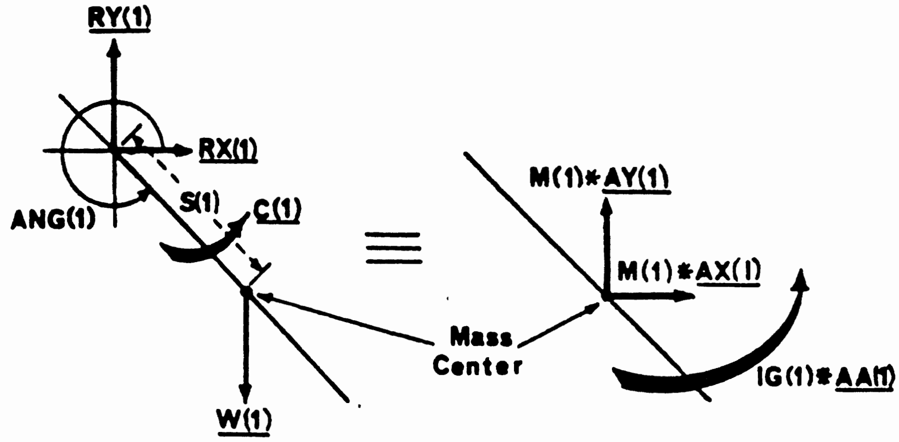
$FX(i), FY(i)$	x and y components of resultant muscle force
$C(i)$	force couple generated by resultant muscle force
$RX(i), RY(i)$	x and y components of the combined resultant muscle and joint reaction forces

in which $i = 1, 3$ to relate the variable designations to particular segments or joints with (1) indicating the foot or ankle; (2), the lower leg or knee; and (3), the thigh or hip joint.

Since the path of the recovery leg is assumed to be restricted to a single plane, three equations of motion can be derived by summing the forces in two orthogonal directions and by summing the moments of force about a given point. Many of these variables in the mechanical analysis including segmental weights, moments of inertia, mass center locations, limb lengths and linear and angular accelerations can either be determined cinematographically or estimated from previous studies. Five unknowns, however, remain in the free body diagram of the foot: the magnitude and direction of the joint reaction force, and the magnitude, direction and point of application of the resultant muscle force. Mathematical manipulations can be employed to combine some of these unknowns into a more manageable number. The resultant muscle force can be replaced by a force of equal magnitude and direction at the joint center and a couple† equivalent to the rotary effect of the original forces about the joint (Figure 4-15). For convenience, the translated muscle force may be indicated by horizontal and vertical components of suitable magnitudes (Figure 4-16). Further, the muscle force components may be combined with those of the joint reaction force (Figure 4-17). Thus, the number of unknowns in the adjusted free body diagram can be reduced to three: the resultant muscular torque, and the magnitude and direction of the combined muscle and reaction force at the joint. Similar adjustments can be made in the free body diagrams of the lower leg and thigh.

In applying the force-mass-acceleration method to derive the equations of motion, the free body diagram of the system is "equated" to a mass-acceleration diagram (Figure 4-18) since both have the same resultant (Meriam, 1966). The mass-acceleration diagram shows the linear acceleration of the mass center (usually in component form) multiplied by the segmental mass as well as a couple equal to the product of the moment of inertia of the segment about an axis through its mass center perpendicular to the plane of motion and the angular acceleration of the segment. While the couple

†The terms couple, torque and moment are used interchangeably to designate the rotational effect of a force with respect to a given point or axis.



Free Body Diagram — Mass-Acceleration Diagram

FIGURE 4-18. *Basis for Deriving Equations of Motion of the Foot.*

can act anywhere in the plane of motion, it is customarily drawn about the mass center.

The three equations of motion of the foot are then obtained by the following:

1. Summing the forces in the horizontal direction and setting them equal to the segmental mass times the horizontal acceleration of the mass center

$$RX(1) = M(1) * AX(1)$$

2. Summing the forces in the vertical direction and setting them equal to the segmental mass times the vertical acceleration of the mass center

$$RY(1) - W(1) = M(1) * AY(1)$$

3. Summing the moments of force about the mass center and setting them equal to the product of the moment of inertia of the segment with respect to its mass center and the angular acceleration of the segment

$$C(1) + RX(1) * S(1) * SIN(ANG(1)) - RY(1) * S(1) * COS(ANG(1)) = IG(1) * AA(1)$$

in which:

$RX(1)$ and $RY(1)$ are the components of the resultant muscle and reaction forces at the ankle joint;

$M(1)$ and $W(1)$ are the mass and weight of the foot respectively;

$AX(1)$ and $AY(1)$ are the components of linear acceleration of the mass center of the foot;
$C(1)$ is the resultant muscular torque or couple at the ankle;
$S(1)$ is the distance from the proximal joint to the mass center of the foot;
$AA(1)$ is the angular acceleration of the foot, often designated by the Greek letter alpha (α);
$ANG(1)$ is the angle of the foot with the right horizontal in a counterclockwise direction measured at the ankle joint; and
$IG(1)$ is the moment of inertia of the foot about its mass center or center of gravity.

Summing the moments about a point that is neither "fixed" nor the center of mass of the segment.

While the moment equation may sometimes be expressed as

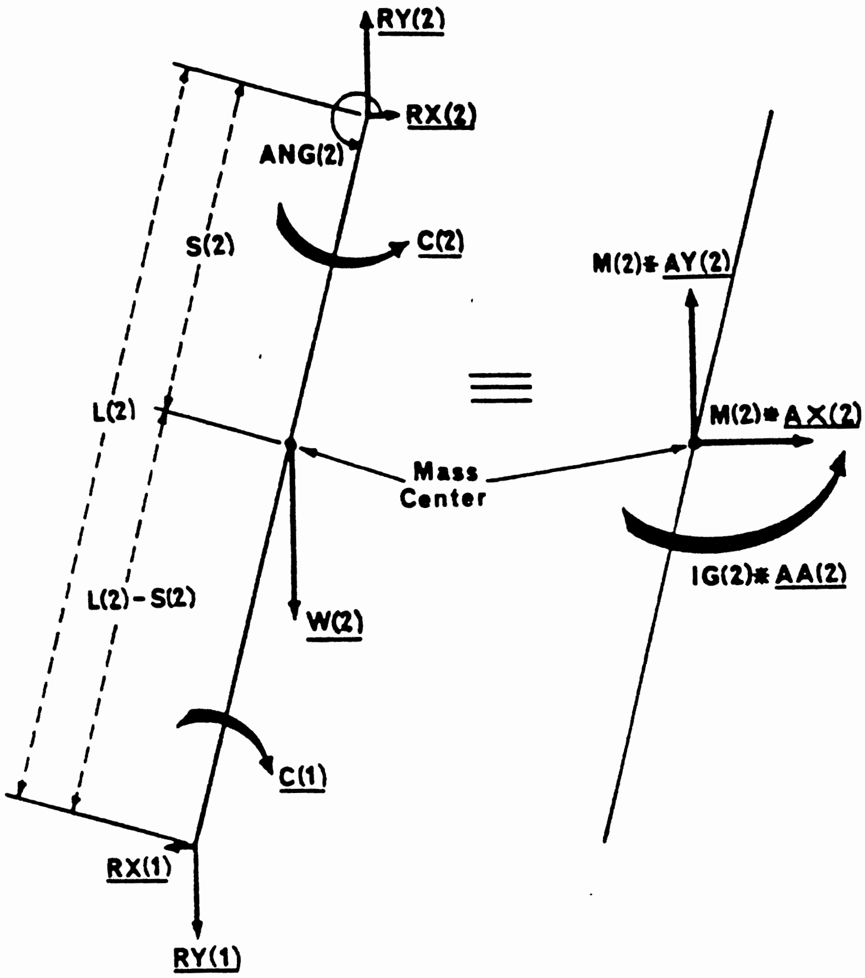
$$\Sigma M_o = I_o \alpha$$

this relationship actually applies only to the special cases in which o is a fixed point or the center of mass of the rigid body. A more general approach which does not require these specific conditions may be used. Moments can be summed about any point on the free body diagram and set equal to the sum of the moments about the same point on the mass-acceleration diagram. Forces and mass-acceleration terms whose lines of action do not pass through the chosen point have a rotational effect or moment about that point. For example, moments could be summed conveniently about the ankle joint, yielding the following equation:

$$C(1) - W(1) \cdot S(1) \cdot COS(ANG(1)) = IG(1) \cdot AA(1) + M(1) \cdot AY(1) \cdot S(1) \cdot COS(ANG(1)) - M(1) \cdot AX(1) \cdot S(1) \cdot SIN(ANG(1))$$

which is equivalent to the moment equation derived earlier.

A careful examination of Figure 4-17 reveals that the reaction force, resultant muscle force and/or resultant muscular torque on either side of the joint are equal in magnitude and opposite in direction. If the foot, lower leg and thigh were to be joined together at any stage of the analysis, the muscle and joint forces at the knee and ankle would cancel one another and would not appear explicitly in the equations. Therefore, the values of $C(1)$, $RX(1)$ and $RY(1)$ determined from the equations of motion for the foot can be substituted into the force-mass-acceleration relationships for the lower leg. Similarly, $C(2)$, $RX(2)$ and $RY(2)$ from the lower leg can be used in the equations of motion for the thigh. It should be realized, however, that such a derivation of the equations of motion assumes that the muscles cross only one joint or that the discrepancies introduced by two joint muscles can be considered negligible in the analysis.



Free Body Diagram — Mass-Acceleration Diagram

FIGURE 4-19. *Basis for Deriving the Equations of Motion of the Lower Leg.*

In deriving the equations of motion for the lower leg and thigh, the same principles and variable designations are employed with the addition of *L* for segment length. The three equations for the lower leg are obtained by the following (Figure 4-19):

1. Summing the forces in the horizontal direction

$$RX(2) - RX(1) = M(2) * AX(2)$$

2. Summing the forces in the vertical direction

$$- RY(1) + RY(2) - W(2) = M(2) * AY(2)$$

3. Summing the moments of force about the mass center of the segment

$$-C(1) + C(2) + RX(1) \cdot (L(2) - S(2)) \cdot SIN(ANG(2)) - RY(1) \cdot (L(2) - S(2)) \cdot COS(ANG(2)) - RY(2) \cdot S(2) \cdot COS(ANG(2)) + RX(2) \cdot S(2) \cdot SIN(ANG(2)) = IG(2) \cdot AA(2)$$

The three equations of motion for the thigh can be developed in a similar manner. When solving these equations for the resultant force components $RX(i)$ and $RY(i)$ and resultant muscle couple $C(i)$, a positive sign indicates that the particular vector is directed in the sense shown on the free body diagram while a negative sign means that the vector acts in the opposite direction.

Utilizing these force-mass-acceleration principles, Dillman (1970) determined the values of the resultant muscular torques at the ankle, knee and hip joints. With these data and a knowledge of the kinematics of the lower limb, he was able to identify the pattern of dominant muscle group activity corresponding to the motion of the recovery leg in sprint running. Other investigators including Elftman (1940), Pearson and his associates (1963) and Plagenhoef (1966, 1971) have also employed this type of approach for studying the dynamics of human performance.

Free Fall Conditions. In sport, it is not uncommon for a performer or his sports implement to experience a condition of non-support or free fall. Some of these activities, which include golf, tennis, badminton, sky diving and ski jumping, are significantly influenced by air resistance. In others, such as those listed in Table 4-4, the effect of air resistance can be disregarded, thereby considerably simplifying the mechanical analysis. Further discussion in this section will be confined to the free fall portion of sports skills in which negligible air resistance can be assumed.

The performance of a skill in an unsupported situation may be classified as general plane motion or general space motion. Thus, the action can be analyzed in terms of the translation of the athlete's mass center and the rotation of the body and its segments. If the movement is of a spatial nature, three components of translation may be considered; one in the vertical direction and two in a horizontal plane at right angles to one another. Likewise rotation is expressed about a vertical or longitudinal body axis as well as about two horizontal axes which usually correspond to the frontal (lateral) and sagittal (anteroposterior) planes of the body. In sport, rotations about the first two are customarily referred to as twisting and somersaulting (Figure 4-20). In aerospace terminology, the three axes are designated yaw, pitch and roll respectively. Although the principal object of