St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Assignment 3
Due November 15, 2021 at 1:15pm

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. Consider the following context-free grammar G, where $V = \{R, S, T, X\}$, $\Sigma_G = \{\mathtt{a}, \mathtt{b}\}$, the start nonterminal is R, and the rule set contains the following rules:

$$\begin{aligned} R &\to XRX \mid S \\ S &\to \mathtt{a}T\mathtt{b} \mid \mathtt{b}T\mathtt{a} \\ T &\to XTX \mid X \mid \epsilon \\ X &\to \mathtt{a} \mid \mathtt{b} \end{aligned}$$

Convert G to an equivalent pushdown automaton $\mathcal{M}$. You do not need to draw the pushdown automaton, you just need to give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

[6 marks] 2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Using the pumping lemma for context-free languages, prove that the following language is not context-free:

$$L = \{\mathtt{a}^i\mathtt{b}^k\mathtt{c}^i\mathtt{d}^i \mid i \geq 1, k \geq 1\}.$$

[6 marks] 3. Consider the Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ where $Q = \{q_0, q_1, q_2, q_{\text{A}}, q_{\text{R}}\}$, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, $\Gamma = \{\mathtt{a}, \mathtt{b}, \sqcup\}$, and δ is defined as follows:

$$\begin{aligned} \delta(q_0, \mathtt{a}) &= (q_1, \mathtt{b}, R); \\ \delta(q_0, \mathtt{b}) &= (q_2, \mathtt{b}, L); \\ \delta(q_0, \sqcup) &= (q_{\text{A}}, \sqcup, L); \\ \delta(q_1, \mathtt{a}) &= (q_2, \mathtt{a}, L); \\ \delta(q_1, \mathtt{b}) &= (q_0, \mathtt{b}, R); \\ \delta(q_1, \sqcup) &= (q_{\text{R}}, \sqcup, L) \\ \delta(q_2, \mathtt{a}) &= (q_2, \mathtt{a}, L); \\ \delta(q_2, \mathtt{b}) &= (q_2, \mathtt{b}, L); \\ \delta(q_2, \sqcup) &= (q_{\text{R}}, \sqcup, R) \end{aligned}$$

(a) Give the sequence of configurations that $\mathcal{M}$ enters when it is given the input word `abab`.

(b) Give the sequence of configurations that $\mathcal{M}$ enters when it is given the input word `abba`.

(c) What language does $\mathcal{M}$ recognize?

[5 marks] 4. (a) By our definition of a Turing machine's transition function, the input head can make one leftward move or one rightward move on each computation step. We could optionally specify a "no move" transition N, where the input head neither moves left nor right (i.e., it stays on the same cell of the input tape). However, such a modification would not affect the recognition power of the Turing machine.

Explain briefly how we could simulate a "no move" transition of the form $\delta(q_i, c) = (q_j, b, N)$ using only leftward and rightward moves.

(b) Consider another variant of our Turing machine model where the transition function is defined as follows:

$$\delta : (Q \setminus \{q_{\text{accept}}, q_{\text{reject}}\}) \times \Gamma \to Q \times \Gamma \times \{R, N\}.$$

This variant machine is only able to move its input head rightward by one cell, or stay where it is (i.e., make "no move").

Does this variant machine recognize the same class of languages as an ordinary Turing machine? If so, explain why. If not, what class of languages does it recognize?

Hint. Starting at the first symbol of the input word, if the input head can only move rightward or make no move, how many times can the Turing machine read each symbol on its tape?