St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Assignment 4
Due December 6, 2021 at 1:15pm

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[4 marks] 1. Consider the following decision problem for regular expressions:

$$IN_{\mathsf{RE}} = \{\langle R, S\rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}.$$

Is IN_{RE} decidable or undecidable? If it is decidable, give a decision procedure to decide the problem. If it is undecidable, give a reduction to prove undecidability.

Hint. You may find the following fact useful: $L(R) \subseteq L(S)$ if and only if $L(R) \cap \overline{L(S)} = \emptyset$.

[4 marks] 2. Consider the following decision problem for context-free grammars:

$$A_{\epsilon\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a context-free grammar that generates the empty word } \epsilon\}.$$

Is $A_{\epsilon\mathsf{CFG}}$ decidable or undecidable? If it is decidable, give a decision procedure to decide the problem. If it is undecidable, give a reduction to prove undecidability.

[6 marks] 3. Recall the emptiness problem for context-free grammars, E_{CFG}. Answer each of the following questions about E_{CFG} and give a brief (1–2 sentence) justification for each answer.

(a) Does there exist a *decidable* language B such that $E_{\mathsf{CFG}} \leq_m B$?

(b) Does there exist an *undecidable* language C such that $C \leq_m E_{\mathsf{CFG}}$?

[6 marks] 4. Recall the universality problem for deterministic finite automata, U_{DFA}. We know that U_{DFA} is decidable. Prove additionally that $U_{\mathsf{DFA}} \in \mathsf{P}$ by giving a polynomial-time algorithm to decide the problem. Give a brief (1–2 sentence) justification of why your algorithm (i) is correct, and (ii) runs in polynomial time.

[5 marks] 5. *Bonus.* A deterministic two-dimensional automaton is a finite automaton that receives as input a two-dimensional word (i.e., an array of symbols from some alphabet Σ) and can move its input head through the input word in any of four directions: upward, downward, leftward, and rightward.

Consider the following decision problem for two-dimensional automata:

$EQ_{\mathsf{2DFA\text{-}4W}} = \{\langle \mathcal{M}_1, \mathcal{M}_2 \rangle \mid \mathcal{M}_1 \text{ and } \mathcal{M}_2 \text{ are deterministic two-dimensional automata and } L(\mathcal{M}_1) = L(\mathcal{M}_2)\}$.

Prove that $EQ_{\mathsf{2DFA\text{-}4W}}$ is undecidable.