St. Francis Xavier University
Department of Computer Science

CSCI 541: Theory of Computing
Assignment 1
Due October 15, 2021 at 9:15am

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

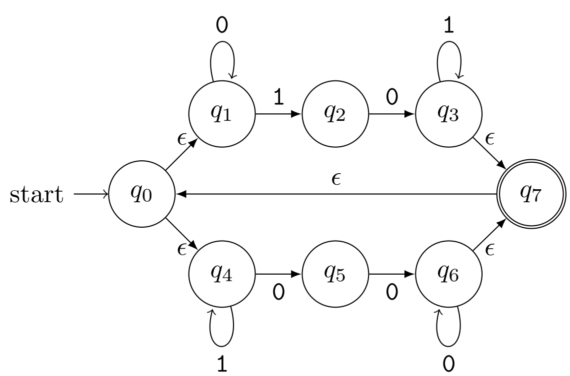
[6 marks] 1. Consider the following language over the alphabet $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$:

$$L = \{w \mid w \text{ begins with } \texttt{a}, \text{ contains zero or more } \texttt{b}\text{s, and ends with either } \texttt{a} \text{ or } \texttt{c}\}.$$

(a) Give a regular expression for the language; that is, show how to define the language using only the empty word ϵ, the symbols a, b, and c, and the operations of union ($\cup$), concatenation ($\cdot$), and Kleene star (*).

(b) Convert your regular expression from part (a) to a finite automaton. Show all your work in addition to giving the finite automaton. You do not need to remove epsilon transitions or determinize the finite automaton.

[6 marks] 2. Consider the following nondeterministic finite automaton $\mathcal{M}$ with epsilon transitions:



(a) Convert $\mathcal{M}$ to a nondeterministic finite automaton without epsilon transitions. Show all your work in addition to giving the finite automaton.

(b) Using your nondeterministic finite automaton from part (a), use the subset construction to complete the table corresponding to the transition function for the equivalent deterministic finite automaton. You do not need to draw the deterministic finite automaton itself.

[8 marks] 3. Using the pumping lemma for regular languages, prove that the following language is not regular:

$$L_{\text{prime}} = \{\mathtt{a}^n \mid n \text{ is a prime number}\}.$$

[6 marks] 4. For each of the following languages over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$, determine whether the language is regular or context-free. If the language is regular, then give a regular expression that represents the language. If the language is context-free, then give a context-free grammar that generates the language.

(a) $L_1 = \{\mathtt{a}^{2i+1}\mathtt{b}^{4m+2} \mid i \geq 0, m \geq 0\} \cup \{\mathtt{c}^{r+1}\mathtt{d}^{3s+1} \mid r \geq 0, s \geq 0\}$.

(b) $L_2 = \{\mathtt{a}^{i}\mathtt{b}^{2k}\mathtt{c}^{k}\mathtt{d}^{3i} \mid i \geq 1, k \geq 1\} \cup \{\mathtt{a}^{r}\mathtt{b}^{2r}\mathtt{c}^{s}\mathtt{d}^{3s} \mid r \geq 1, s \geq 1\}$.

[4 marks] 5. Consider the following context-free language over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$:

$$L = \{\mathtt{a}^{2i}\mathtt{b}^{2k}\mathtt{c}^{i} \mid i \geq 1, k \geq 0\}.$$

(a) Give a context-free grammar that generates L.

(b) Construct a pushdown automaton that recognizes L.