

St. Francis Xavier University
Department of Computer Science
CSCI 541: Theory of Computing
Assignment 2
Due November 26, 2021 at 9:15am

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

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- [4 marks] 1. Consider the following context-free grammar G , where $V = \{S, A\}$, $\Sigma_G = \{0, 1\}$, the start nonterminal is S , and the rule set contains the following rules:

$$S \rightarrow AS \mid \epsilon$$
$$A \rightarrow 0A1 \mid A1 \mid 01$$

Convert G to an equivalent pushdown automaton \mathcal{N} . You do not need to draw the pushdown automaton, you just need to give each component of the tuple $\mathcal{N} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

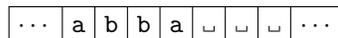
- [10 marks] 2. Let $\Sigma = \{a, b, c, d\}$. For each of the following languages, determine whether or not the language is context-free. If the language is context-free, then give a context-free grammar that generates the language. If the language is not context-free, then prove this using the pumping lemma.

(a) $L_1 = \{a^i b^k c^{2i} d^{3m} \mid i \geq k \geq 1, m \geq 1\}$.

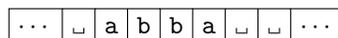
(b) $L_2 = \{a^{2i} b^i c^{3k} d^k \mid i \geq 1, k \geq 1\}$.

- [6 marks] 3. Construct a single-tape deterministic Turing machine \mathcal{M} that performs the following “right shift” operation: given a word $w \in \{a, b\}^*$, \mathcal{M} shifts the entire input word rightward by one cell on its input tape. After performing this shift, \mathcal{M} halts and accepts.

For example, if \mathcal{M} is given the input word **abba**, its tape at the beginning of the computation will look like



and, after \mathcal{M} halts and accepts, its tape will look like



You should give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$. You may optionally draw the Turing machine to illustrate your construction.

- [5 marks] 4. Consider the problem of testing whether a deterministic finite automaton \mathcal{B} and a regular expression R are equivalent. Express this decision problem as a language $EQ_{\text{DFA,RE}}$, and show that the problem is decidable.

- [5 marks] 5. A *useless state* in a Turing machine is a state that is never visited during the Turing machine's computation on any input word. Consider the following decision problem:

$$USELESS_{\text{TM}} = \{\langle \mathcal{M}, q \rangle \mid q \text{ is a useless state in the Turing machine } \mathcal{M}\}.$$

Show that the problem $USELESS_{\text{TM}}$ is undecidable.

Hint. Reduce from another undecidable problem for Turing machines.