St. Francis Xavier University
Department of Computer Science

CSCI 541: Theory of Computing
Assignment 2
Due November 26, 2021 at 9:15am

Assignment Regulations.

- This assignment may be completed individually or in a group of up to four people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[4 marks] 1. Consider the following context-free grammar G, where $V = \{S, A\}$, $\Sigma_G = \{\mathtt{0}, \mathtt{1}\}$, the start nonterminal is S, and the rule set contains the following rules:

$$S \to AS \mid \epsilon$$
$$A \to \mathtt{0}A\mathtt{1} \mid A\mathtt{1} \mid \mathtt{01}$$

Convert G to an equivalent pushdown automaton $\mathcal{N}$. You do not need to draw the pushdown automaton, you just need to give each component of the tuple $\mathcal{N} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

[10 marks] 2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. For each of the following languages, determine whether or not the language is context-free. If the language is context-free, then give a context-free grammar that generates the language. If the language is not context-free, then prove this using the pumping lemma.

(a) $L_1 = \{\mathtt{a}^i\mathtt{b}^k\mathtt{c}^{2i}\mathtt{d}^{3m} \mid i \geq k \geq 1, m \geq 1\}$.

(b) $L_2 = \{\mathtt{a}^{2i}\mathtt{b}^i\mathtt{c}^{3k}\mathtt{d}^k \mid i \geq 1, k \geq 1\}$.

[6 marks] 3. Construct a single-tape deterministic Turing machine $\mathcal{M}$ that performs the following "right shift" operation: given a word $w \in \{\mathtt{a}, \mathtt{b}\}^*$, $\mathcal{M}$ shifts the entire input word rightward by one cell on its input tape. After performing this shift, $\mathcal{M}$ halts and accepts.

For example, if $\mathcal{M}$ is given the input word `abba`, its tape at the beginning of the computation will look like

···	a	b	b	a	␣	␣	␣	···

and, after $\mathcal{M}$ halts and accepts, its tape will look like

···	␣	a	b	b	a	␣	␣	···

You should give each component of the tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$. You may optionally draw the Turing machine to illustrate your construction.

[5 marks] 4. Consider the problem of testing whether a deterministic finite automaton $\mathcal{B}$ and a regular expression R are equivalent. Express this decision problem as a language $EQ_{\mathsf{DFA,RE}}$, and show that the problem is decidable.

[5 marks] 5. A *useless state* in a Turing machine is a state that is never visited during the Turing machine's computation on any input word. Consider the following decision problem:

$$USELESS_{\mathsf{TM}} = \{\langle \mathcal{M}, q\rangle \mid q \text{ is a useless state in the Turing machine } \mathcal{M}\}.$$

Show that the problem $USELESS_{\mathsf{TM}}$ is undecidable.

Hint. Reduce from another undecidable problem for Turing machines.