St. Francis Xavier University
Department of Computer Science

CSCI 356: Theory of Computing
Assignment 3
Due November 21, 2022 at 1:15pm

Assignment Regulations.

- This assignment must be completed individually.
- Please include your full name and email address on your submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[8 marks] 1. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Consider the language

$$L_1 = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^{2j}\mathtt{d}^{2i} \mid i \geq 1, j \geq 1\}.$$

Construct a pushdown automaton that recognizes the language L_1, then give a trace of the behaviour of your pushdown automaton on the input word `abbccccdd`. Your trace should list, at each step of the computation: the current state of the pushdown automaton, the current stack contents, and the remaining input word symbols.

[6 marks] 2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Using the pumping lemma for context-free languages, prove that the following language is not context-free:

$$L_2 = \{\mathtt{a}^k\mathtt{b}^{2k}\mathtt{c}^i \mid k \geq i \geq 1\}.$$

[4 marks] 3. Consider the Turing machine $\mathcal{M}_1 = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ where $Q = \{q_0, q_1, q_2, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, $\Gamma = \{\mathtt{a}, \mathtt{b}, \sqcup\}$, and δ is defined as follows:

$$\begin{aligned}
\delta(q_0, \mathtt{a}) &= (q_1, \mathtt{b}, R);\\
\delta(q_0, \mathtt{b}) &= (q_2, \mathtt{b}, L);\\
\delta(q_0, \sqcup) &= (q_{\text{accept}}, \sqcup, L);\\
\delta(q_1, \mathtt{a}) &= (q_2, \mathtt{a}, L);\\
\delta(q_1, \mathtt{b}) &= (q_0, \mathtt{b}, R);\\
\delta(q_1, \sqcup) &= (q_{\text{reject}}, \sqcup, L)\\
\delta(q_2, \mathtt{a}) &= (q_2, \mathtt{a}, L);\\
\delta(q_2, \mathtt{b}) &= (q_2, \mathtt{b}, L);\\
\delta(q_2, \sqcup) &= (q_{\text{reject}}, \sqcup, R)
\end{aligned}$$

(a) Give the sequence of configurations that $\mathcal{M}_1$ enters when it is given the input word `ababab`.

(b) Give the sequence of configurations that $\mathcal{M}_1$ enters when it is given the input word `abbaab`.

(c) What language does $\mathcal{M}_1$ recognize?

[7 marks] 4. Imagine a Turing machine $\mathcal{M}_2$ that can apply the following "right shift" operation to its input word:

- $\mathcal{M}_2$ receives as input a word over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.
- $\mathcal{M}_2$ shifts the entire input word on its tape to the right by one tape cell. After this shift is performed, the tape cell that originally contained the first symbol of the input word will contain a blank space.
- $\mathcal{M}_2$ halts in its accepting state after shifting all input word symbols in this manner.

As an example, if $\mathcal{M}_2$ receives the word `aababb` as input, then it will begin its computation with the tape contents

$$\cdots \mathtt{aababb}\sqcup \cdots$$

and end its computation with the tape contents

$$\cdots \sqcup \mathtt{aababb} \cdots$$

(Other blank spaces to the left and to the right of the input word are omitted.)

Construct the Turing machine $\mathcal{M}_2$ that is capable of applying this "right shift" operation to its input word, and give a brief ($\sim$1 paragraph) justification of how your Turing machine works.