St. Francis Xavier University
Department of Computer Science

CSCI 541: Theory of Computing
Assignment 1
Due October 11, 2022 at 8:15am

Assignment Regulations.

- This assignment may be completed individually or in a group of two people. If you are collaborating on an assignment as a group, your group must submit exactly one joint set of answers.
- Please include your full name and email address on your submission. For groups, every member must include their full name and email address on the joint submission.
- You may either handwrite or typeset your submission. If your submission is handwritten, please ensure that the handwriting is neat and legible.

[6 marks] 1. For each of the following languages over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$, show how to define the language using only the empty word ϵ, the symbols `0` and `1`, and the operations of union ($\cup$), concatenation ($\cdot$), and Kleene star (*).

(a) $L_1 = \{w \mid w \text{ starts with } \mathtt{0110} \text{ or ends with } \mathtt{1001}\}$.

(b) $L_2 = \{w \mid w \text{ is of even length}\}$.

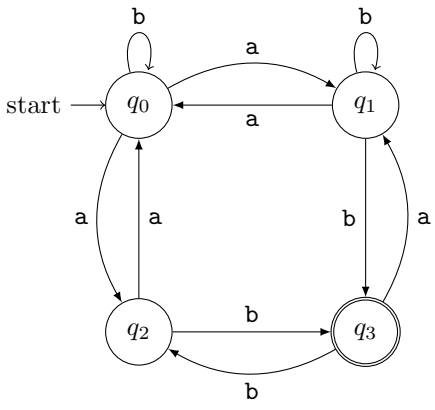
(c) $L_3 = \{w \mid w \text{ begins with } \mathtt{0}, \text{ ends with } \mathtt{0}, \text{ and does not contain } \mathtt{010} \text{ as a subword}\}$.

[6 marks] 2. Consider the following regular language over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$:

$$(\mathtt{01}^* \cup \mathtt{10})^*$$

Construct a finite automaton recognizing this regular language. Show all your work in addition to giving the finite automaton. You do not need to remove epsilon transitions or determinize the finite automaton.

[8 marks] 3. Given the following nondeterministic finite automaton $\mathcal{M}$, convert it to a deterministic finite automaton $\mathcal{M}'$ recognizing the same language. Show all your work in addition to giving the deterministic finite automaton.



[6 marks] 4. Let $\Sigma = \{\texttt{(}, \texttt{)}\}$. A word w over Σ contains balanced parentheses if every opening parenthesis is matched by a closing parenthesis and each pair of parentheses is correctly nested. Thus, the words `()`, `()(())`, and `(((())())())` all contain balanced parentheses, but the words `(()()(()` and `)))(())((((())` do not.

Using the pumping lemma for regular languages, prove that the following language is not regular:

$$L_{()} = \{w \mid w \text{ contains balanced parentheses}\}.$$

[4 marks] 5. Is the following language over $\Sigma = \{\texttt{a}, \texttt{b}\}$ a regular language? If it is, give a regular expression corresponding to the language or a finite automaton recognizing the language. If it is not, use the pumping lemma for regular languages to prove this.

$$L = \{uww^{\mathrm{R}}v \mid u, w, v \in \{\texttt{a}, \texttt{b}\}^{+}\}$$

Hint. Be careful! This language can be deceiving at first glance.